Use of State-Dependent Strength in Estimating End Bearing Capacity of Piles in Sand

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Abstract: The pressure and density dependence of the shear strength of sand poses a tricky problem in pile foundation design. In this study, a correlation is suggested to link the effective friction angle of sand with its initial confining pressure and relative density, and a simple approach incorporating this correlation is presented for predicting pile end bearing capacity. Assessment of the approach against pile load tests shows reasonably good agreement between predictions and measurements. It is also shown that the effect of the state-dependent strength is particularly important in cases where long piles are installed in dense sand deposits and the use of critical state friction angle will produce a conservative prediction in such cases.

DOI: 10.1061/(ASCE)1090-0241(2008)134:7(1010)

CE Database subject headings: Bearing capacity; Piles; Pile foundations; Sand; Shear strength.

Introduction

The bearing capacity of displacement piles in sand is an area involving great uncertainty and empiricism (Randolph 2003; Yang 2006). Apart from the empirical approaches based on CPT or SPT data, a common approach of theoretical nature to the determination of the ultimate end bearing resistance of piles in sand, q_b, makes use of the relationship:

$$q_b = N_q \sigma'_{v0} \tag{1}$$

where σ'_{v0}=vertical effective stress at the level of the pile tip and N_q is known as the bearing capacity factor, which is a function of the effective friction angle of the sand, ϕ'. Various bearing capacity theories have been proposed for the determination of N_q, among them is one proposed by Berezantzev et al. (1961) appears to be most widely used in practice (Poulos and Davis 1980; Tomlinson 1994; GEO 2006).

Note that the value of N_q is very sensitive to the variation of ϕ'. Therefore, it is crucial to choose an appropriate value of the effective friction angle in the design. Numerous experiments in the literature have shown that the stress–strain–strength behavior of sand is complicated and influenced by many factors. When subjected to shearing, a loose sand contracts and a dense sand dilates. Whether a sand is in a loose or dense state not only depends on the relative density but also on the confining pressure (Yang and Li 2004). Moreover, both the dense and loose specimens of a sand will eventually reach an ultimate state, in which the mean effective stress, the deviatoric stress and the volumetric strain no longer change [Fig. 1(a)]. This ultimate state is known as the critical state which forms a unique critical state line in the void ratio–mean effective stress plane.

Bolton (1986) conducted a comprehensive review of experimental data on shear strength of sands and suggested an empirical correlation that has been widely used in the design of pile foundations (e.g., Fleming et al. 1992):

$$\phi'_p - \phi'_{cs} = 3[D_r(10 - \ln p'_f) - 1] \tag{2}$$

where ϕ'_p=effective friction angle at peak strength; ϕ'_{cs}=critical state friction angle; and D_r=relative density.

It is worth noting here that p'_f=mean effective stress *at failure* (kPa) rather than at the initial state. From the viewpoint of constitutive modeling, it is not considered appropriate to treat the failure stress as a constant, input parameter. From the viewpoint of practical application, this treatment may render it inconvenient to use Eq. (2) in foundation design. To tackle this problem, an improvement of Bolton's correlation is made in this study such that the effective friction angle at peak strength is linked with the *initial* mean effective stress. A simple and practical procedure is then presented that incorporates the new correlation for predicting the end bearing capacity of piles in sand. The feasibility of the procedure is evaluated by using results from full-scale pile load tests and the impact of taking into account the state-dependent strength in design is examined.

State-Dependent Shear Strength

The correlation expressed by Eq. (2) was established using data from triaxial compression tests. In a conventional triaxial setting, the relationship between the mean effective stress, p', and the deviatoric stress, q, can be expressed as follows [Fig. 1(b)]:

$$\frac{q}{p' - p'_0} = 3 \tag{3}$$

where p'_0=initial mean effective stress. The stress state at failure satisfies

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Note. Discussion open until December 1, 2008. Separate discussions must be submitted for individual papers. To extend the closing date by one month, a written request must be filed with the ASCE Managing Editor. The manuscript for this technical note was submitted for review and possible publication on June 17, 2007; approved on October 3, 2007. This technical note is part of the ***Journal of Geotechnical and Geoenvironmental Engineering***, Vol. 134, No. 7, July 1, 2008. ©ASCE, ISSN 1090-0241/2008/7-1010–1014/$25.00.

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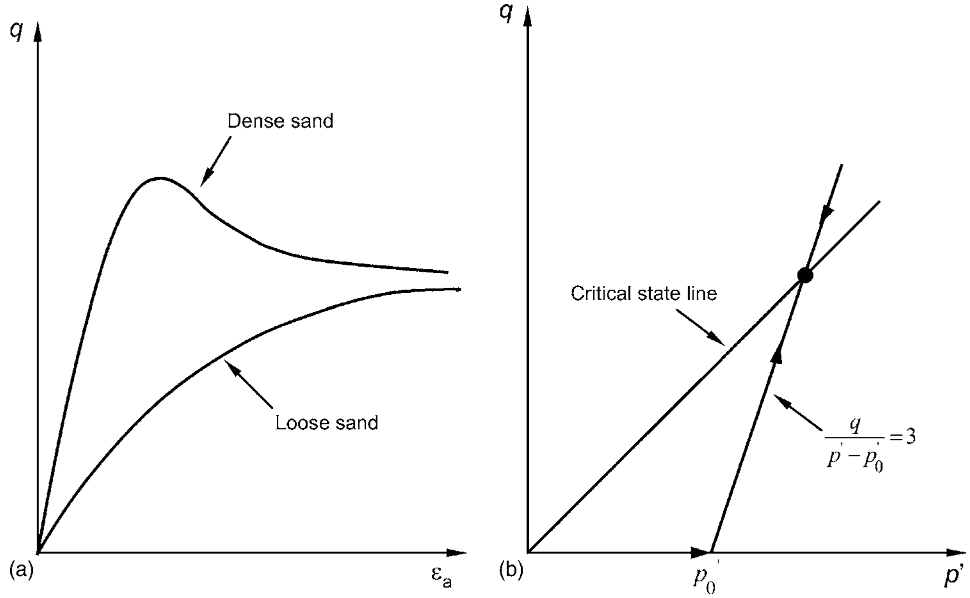

Fig. 1. Typical response of sand in triaxial compression tests: (a) stress–strain behavior; (b) stress path

$$M_p = \frac{q}{p'} = \frac{6 \sin \phi'_p}{3 - \sin \phi'_p} \tag{4}$$

By combining Eqs. (3) and (4), the mean effective stress at failure can be related to the initial mean effective stress in the form as

$$p'_f = \frac{3p'_0}{3 - M_p} \tag{5}$$

Introducing Eq. (5) into Eq. (2) gives the effective friction angle as a function of the initial mean effective stress:

$$\phi'_p - \phi'_{cs} = 3D_r\left(10 - \ln\left(p'_0 \Bigg/ \left(1 - \frac{2 \sin \phi'_p}{3 - \sin \phi'_p}\right)\right)\right) - 3 \tag{6}$$

Using Eq. (6), $(\phi'_p - \phi'_{cs})$ is calculated as a function of the initial mean effective stress at several different relative densities [Fig. 2(a)]. Note that the lower limit of the effective friction angle is ϕ'_{cs}, which is essentially relevant to sand mineralogy and angularity and has typical values of 30–34°. To validate the derived correlation, new data from triaxial compression tests by Maeda and Miura (1999a,b) is compiled in Table 1 and shown in Fig. 2(b) along with the predictions produced by Eq. (6). A reasonable agreement can be observed. As an example, Figs. 3 and 4 present the typical responses of a strongly dilatant sand and a medium dilatant sand that have been used to derive the data points in Fig. 2(b).

End Bearing Capacity of Piles

A simple procedure for predicting the end bearing capacity of piles that takes into account the state-dependent strength of sand is proposed as follows. First, the initial confining pressure at the level of pile tip, p'_0, is assumed to be the effective overburden pressure σ'_{v0}. Given the relative density and critical state angle,

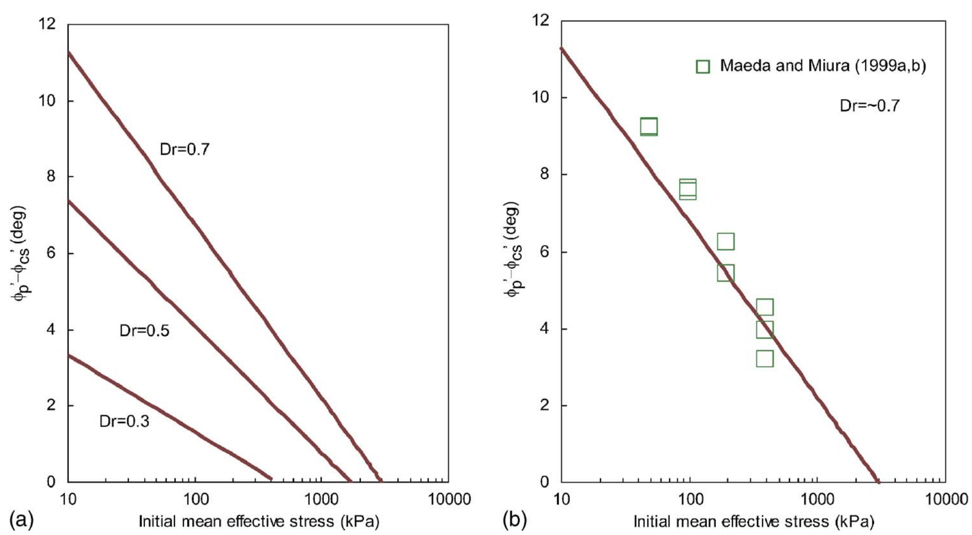

Fig. 2. State-dependent effective friction angle: (a) prediction; (b) prediction versus experimental data

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Table 1. Triaxial Tests Used in This Study

Test	Sand	D_{50} (mm)	U_c	D_r	p_0' (kPa)
1	CA106	0.089	1.19	0.70	98
2	CA106	0.089	1.19	0.70	196
3	CA106	0.089	1.19	0.72	392
4	CA250	0.230	1.09	0.69	392
5	CA300	0.274	1.10	0.69	392
6	SO250	0.230	1.09	0.69	98
7	SO300	0.274	1.10	0.69	49
8	SO300	0.274	1.10	0.69	196
9	SO850	0.714	1.19	0.70	49

Note: Data courtesy of K. Maeda.

the peak friction angle ϕ_p' can then be determined by using Eq. (6). This further allows the earth pressure coefficient K_0 to be estimated (in the first instance the soil is assumed herein to be normally consolidated):

$$K_0 = 1 - \sin \phi_p' \quad (7)$$

The confining pressure p_0' is now updated as

$$p_0' = \frac{1 + 2K_0}{3} \sigma_{v0}' \quad (8)$$

The previous steps are repeated until the values of p_0' and ϕ_p' are compatible. Then the value of N_q is determined by entering the compatible friction angle into the curve of Berezantzev et al. (1961) and the end bearing resistance q_b is calculated.

To evaluate the feasibility and accuracy of the proposed method, a number of load tests on piles in sand are collected and

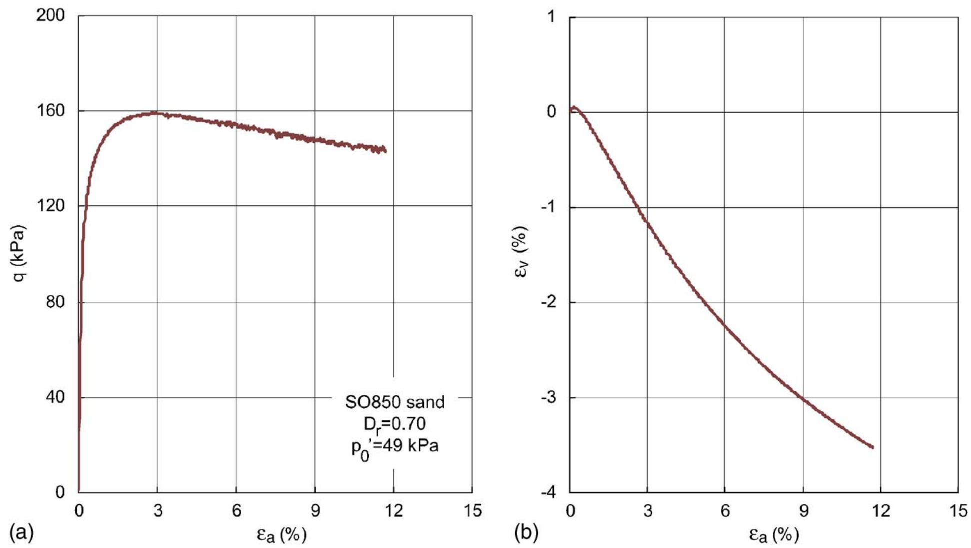

Fig. 3. Response of strong dilatant sand in triaxial compression (data courtesy of K. Maeda): (a) stress–strain behavior; (b) volumetric behavior

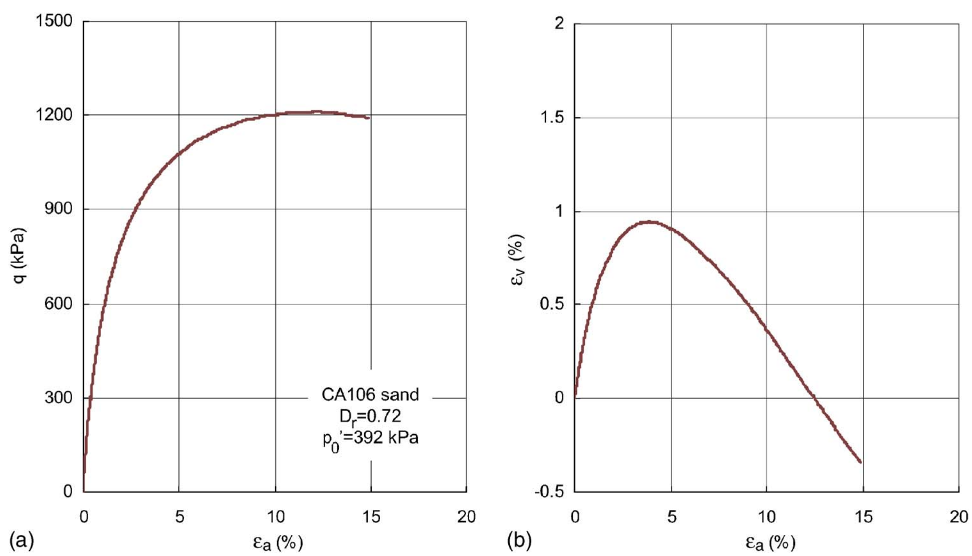

Fig. 4. Response of a medium dilatant sand in triaxial compression (data courtesy of K. Maeda): (a) stress–strain behavior; (b) volumetric behavior

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Table 2. Predicted and Measured End Bearing Resistance of Piles

Source	Test	Pile type	Pile length (m)	Pile diameter (m)	σ'_{v0} at tip (kPa)	D_r	ϕ'_{cs}	Measured q_b (MPa)	Predicted q_b (MPa)
Altaee et al. (1992)	1–1	Driven concrete	11	0.285	153	0.40	30	6.21	8.12
	2	Driven concrete	15	0.285	192	0.35	30	7.52	8.67
Chow (1997)	DK1/L1C	Jacked steel	7.4	0.102	0.2	0.77	31	11.85	18.04
	DK2/L1C	Jacked steel	5.96	0.102	88	0.63	31	10.85	11.41
BCP (1971)	1C	Jacked steel	11	0.2	140	0.86	30	26.08	23.10
	6C	Driven steel	11	0.2	140	0.86	30	20.37	23.10

Note: All test piles were closed-ended.

Table 3. Effect of State-Dependent Strength on Pile End Bearing Capacity

			Method 1		Method 2		Method 3	
Source	Test	Measured	Predicted	Measured/ predicted (%)	Predicted	Measured/ predicted (%)	Predicted	Measured/ predicted (%)
Altaee et al. (1992)	1–1	6.21	8.12	76	5.05	123	9.17	68
	2	7.52	8.67	87	6.33	119	9.59	78
Chow (1997)	DK1/L1C	11.85	18.04	66	3.99	297	24.83	48
	DK2/L1C	10.85	11.41	95	3.44	315	14.52	75
BCP (1971)	1C	26.08	23.10	113	4.62	565	32.86	79
	6C	20.37	23.10	88	4.62	441	32.86	62
Mean		—	—	88	—	310	—	68
Standard deviation		—	—	16	—	175	—	12

Note: Method 1=proposed method; Method 2=use of ϕ'_{cs} for N_q; Method 3=incorrect use of Bolton's relation.

the predictions are compared with the test results in Table 2. The information on relative density and effective overburden pressure given in Table 2 is derived from Chow (1997). The measured values of end bearing resistance are thought to represent plunging failure (White and Bolton 2005). As ϕ'_{cs} values for the tests of Altaee et al. (1992) and BCP (1971) are not available, a typical value for silty sand (30°) is assumed here.

As shown in Table 2, the proposed method is able to provide a reasonably good prediction of the end bearing capacity of the test piles. The mean value of the ratio between measurement and prediction is 0.88, with the standard deviation of 0.16. Note that the simple method only involves a few relevant parameters that can be assessed with relative ease.

Some practicing engineers may opt to choose the critical state friction angle ϕ'_{cs} in foundation design. In order to examine the influence of state-dependent strength in pile foundation design, prediction has also been made for the load tests in Table 2 by simply using Eq. (1) and the critical state friction angle (referred to as Method 2 hereafter). The results are given in Table 3, together with the values of the measurement-to-prediction ratio. Generally, the prediction is poor and very conservative.

Another point worth mentioning here is that the correlation proposed by Bolton (1986) has been occasionally used by mistake. That is, the mean effective stress at failure in Eq. (2) was regarded as the initial mean stress. If the procedure involved the incorrect use of Bolton's relation (referred to as Method 3) rather than Eq. (6), an overestimate of pile end bearing capacity was obtained, as shown in Table 3 for the same set of pile load tests. An alternative view of the performance of the three methods in predicting the end bearing resistance of the test piles is given in Fig. 5.

In order to better view the impact of properly taking account of the state-dependent strength, the end bearing capacity of a pile in a hypothesized, homogeneous sand deposit (D_r=0.3) is evaluated using the three methods and shown as a function of the pile embedment depth in Fig. 6(a), and a parallel case is analyzed for a very dense deposit having D_r=0.8 [Fig. 6(b)]. In the calculation the critical state friction angle is assumed to be 30° and the effective unit weight is assumed to be 10 kN/m^3. It is observed that the impact is related to the density of sand and the embedment length of the pile. The impact tends to be more significant when the sand is in a dense state and the pile embedment is large.

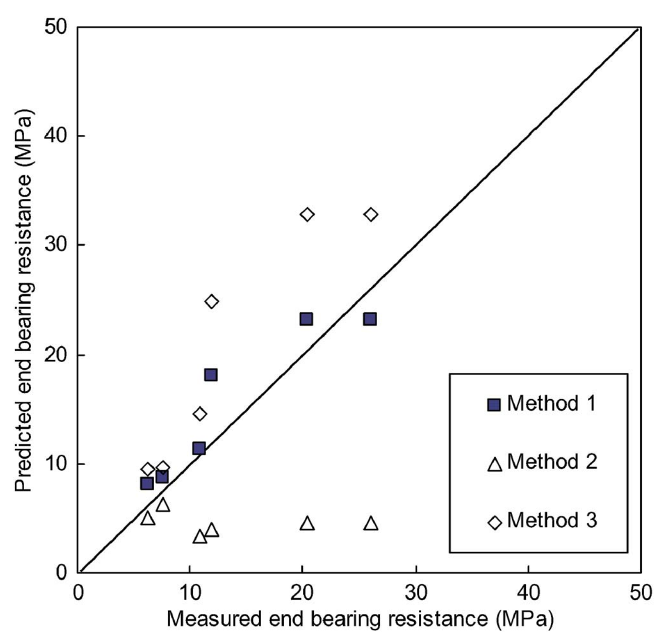

Fig. 5. Predicted and measured end bearing resistance of test piles

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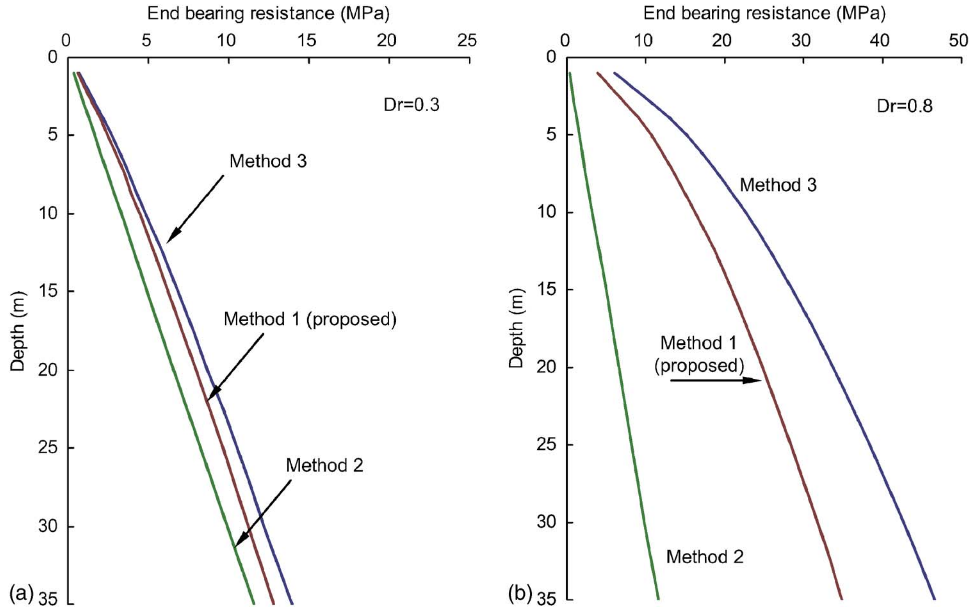

Fig. 6. Predicted end bearing resistance of piles in (a) loose sand and (b) dense sand

Concluding Remarks

A correlation has been suggested between the effective friction angle of sand and its initial confining pressure and relative density for the purpose of facilitating the use of state-dependent strength in foundation design. A simple procedure incorporating the correlation has been presented for predicting the end bearing capacity of piles in sand. The comparison of the predictions produced by the proposed method with the results of pile load tests shows reasonably good agreement. The study also shows that the effect of the state-dependent strength is particularly important in cases where long piles are installed in dense sand deposits. The use of critical state friction angle will produce a conservative prediction whereas the incorrect use of the Bolton's correlation will generally give an overestimate.

Acknowledgments

The writers wish to thank Dr. K. Maeda of Nagoya Institute of Technology for providing the experimental data that were used in validation of the proposed correlation. The work described in this note was partly supported by a grant from the Research Committee of the University of Hong Kong.

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