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TECHNICAL NOTE

Analytical study of saturation effects on seismic vertical amplification of a soil layer

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An analysis is presented of saturation effects on the amplification of a single soil layer to vertical earthquake excitation. Rigorous solutions for various responses are derived based on Biot's theory and the concept of homogeneous pore fluid. The transfer function of the layer is described as a function of degree of saturation, soil properties, layer thickness and loading frequency. Numerical examples are given to illustrate the influence of saturation on the motion amplification. The present study indicates that partial saturation of soils may always lead to a larger amplification compared with a full saturation model, and consequently provides a physical insight into an array observation on vertical site amplification during the Kobe earthquake of 1995.

KEYWORDS: dynamics; vibration; partial saturation; earthquakes.

INTRODUCTION

Strong ground motions were recorded at a reclaimed island, Kobe, Japan, by a downhole array during the Kobe earthquake of 1995. Fig. 1 shows the distribution of recorded peak accelerations with depth in three components. It is of interest to note that, while peak accelerations in both horizontal components (east–west and north–south respectively) were reduced as seismic waves travelling from bottom to surface, the vertical motion was greatly amplified at the surface, with a peak value as large as 1·5–2 times the horizontal components. So far, considerable interest has been paid to the horizontal motion associated with the propagation of shear waves (e.g. Aguirre & Irikura, 1997; Kokusho & Matsumoto, 1998; Yang *et al.*, 2000). These studies concluded that the reduction of horizontal motion was associated with substantial soil non-linearity induced by liquefaction in surface reclaimed layers. However, available analysis of the large amplification in vertical motion is quite limited, which is surprising in view of its unusual feature, as mentioned previously.

It is supposed that the large amplification in the vertical component may involve mainly the propagation of a compression wave (*P* wave) on which the non-linear soil effect is very small (Yang & Sato, 2000a). At the array site considered, the average *P*-wave velocity measured is around 590 m/s for the reclaimed surface layers above the depth of 12·6 m, while it increases significantly to about 1500 m/s at deeper depths, indicating a strong contrast in *P*-wave velocity. It is further considered that the low velocity of the *P* wave may be related to partial saturation of soils, because experimental results have revealed that partial saturation may significantly reduce the velocity of the *P* wave (Allen *et al.*, 1980; Sills *et al.*, 1991; Wheeler *et al.*, 1991). In certain situations surface soils below the water table may not be fully water saturated although the degree of saturation is very high. For example, incomplete saturation may be caused by fluctuating water tables, flooding, or recharge of groundwater. In particular, the situation of partial saturation is a typical case for offshore soils (Sills *et al.*, 1991).

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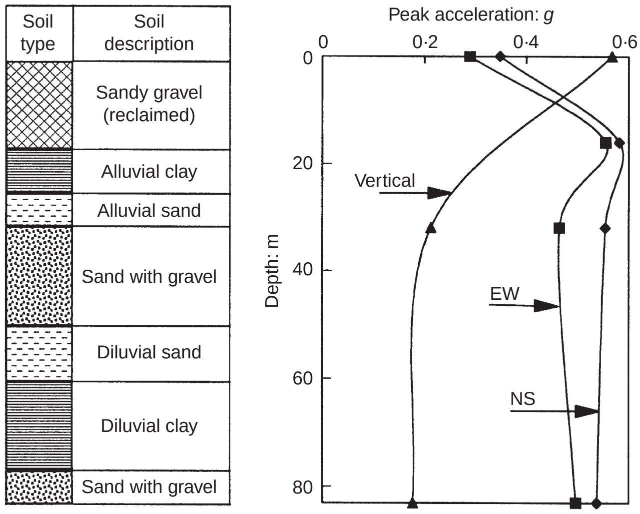

Fig. 1. Distribution of recorded peak accelerations with depth in three components

In this note, the influence of saturation on the amplification of soils to vertical earthquake motion is studied analytically for a simplified problem yet based on multi-phase theory, with the main purpose being to provide a physical insight into the array observation from the Kobe earthquake as mentioned previously. The problem under consideration corresponds to a soil deposit of finite thickness underlain by rigid bedrock, on which steady-state vertical displacement excitation acts. Apparently, this model also represents the case of a shaking table in the laboratory, for which displacement excitation is specified at a rigid base. The soil is treated herein as a partially water saturated material with a small amount of air inclusions. Based on the concept of homogeneous pore fluid and Biot's theory (Biot, 1956), rigorous solutions for various responses such as displacement, stress and pore pressure are derived. Numerical examples are given to illustrate the influence of saturation on the motion amplification.

THEORETICAL FORMULATION

The model considered is shown in Fig. 2. The steady-state displacement excitation is specified in vertical direction at the base of the soil deposit of thickness *L*. The surface of the layer is treated as stress-free and free draining, whereas the base is assumed as impermeable and rigid. The soils are assumed to be highly water saturated, for which the relative proportions of constituent volumes are characterized by porosity *n* and the degree of saturation S_r as

$$n = \frac{V_v}{V_t} \qquad S_r = \frac{V_w}{V_v} \tag{1}$$

where V_v is pore volume, V_w is the volume of pore water, and V_t is total volume. In the case of high saturation (e.g. $S_r > 95\%$), one may treat the air–water mixture as a homogeneous pore fluid by assuming the air is embedded in the form of bubbles. The bulk modulus of the homogeneous fluid, K_f,

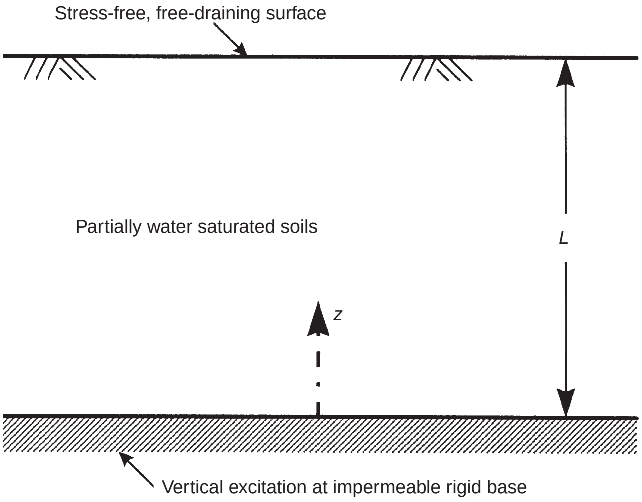

Fig. 2. A soil layer subjected to vertical earthquake excitation at rigid base

depends approximately on the degree of saturation S_r as (Verruijt, 1969)

$$K_f = \frac{1}{1/K_w + (1 - S_r)/p_a} \tag{2}$$

where K_w is the bulk modulus of pore water and p_a is the absolute fluid pressure. It can readily be shown that even a very small amount of air in soil will drastically reduce the bulk modulus of fluid.

Based on Biot's theory and the concept of homogeneous pore fluid, the constitutive equations for macro-isotropic, poroelastic materials with compressible constituents can be expressed as

$$\sigma_{ij} = \lambda e \delta_{ij} + 2\mu\varepsilon_{ij} - \alpha\delta_{ij} p_f \tag{3}$$

$$p_f = M\zeta - \alpha M e \tag{4}$$

where σ_{ij} is total-stress tensor, p_f is pore pressure, δ_{ij} is the Kronecker delta, and

$$\varepsilon_{ij} = \frac{1}{2}(u_{i,j} + u_{j,i}) \tag{5}$$

in which u_i and w_i are displacement vectors of solid skeleton and fluid, respectively; $e = u_{ij}$ and $\zeta = -w_{i,j}$ indicate volumetric strains of soil skeleton and fluid, respectively; and λ and μ are Lame constants, α, M are the parameters accounting for the compressibility of the grains and the fluid. They can be given as

$$\alpha = 1 - \frac{K_b}{K_s} \qquad M = \frac{K_s^2}{K_d - K_b}$$

$$K_d = K_s\left[1 + n\left(\frac{K_s}{K_f} - 1\right)\right] \tag{6}$$

where K_s, K_b and K_f are the bulk moduli of solid grains, solid skeleton and pore fluid respectively. For incompressible solid grains only $\alpha = 1$, while for both grains and pore fluid incompressible, $\alpha = 1$ and $1/M = 0$. The inclusion of the compressibility of constituents is one important feature of Biot's theory, although it is usually simplified in soil mechanics. Mathematically, by introducing the incompressibility of solid grains, equation (3) may be reduced to the well-known effective stress by Terzaghi, as shown in Lade & de Boer (1997).

For the case of one-dimensional wave motion considered, the governing equations can be conveniently given from the general three-dimensional forms (Yang & Sato, 1998) as

$$(\lambda_c + 2\mu)\frac{\partial^2 u}{\partial z^2} + \alpha M\frac{\partial^2 w}{\partial z^2} = \rho\frac{\partial^2 u}{\partial t^2} + \rho_f\frac{\partial^2 w}{\partial t^2} \tag{7}$$

$$\alpha M\frac{\partial^2 u}{\partial z^2} + M\frac{\partial^2 w}{\partial z^2} = \rho_f\frac{\partial^2 u}{\partial t^2} + m\frac{\partial^2 w}{\partial t^2} + b\frac{\partial w}{\partial t} \tag{8}$$

where $\lambda_c = \lambda + \alpha^2 M$; ρ and ρ_f are the mass densities of the unit and pore fluid respectively; the parameter m describes the mass coupling, and in most cases is given as $m = \rho_f/n$; the parameter b accounts for the viscous coupling due to relative fluid motion, which is defined as η/K; η is fluid viscosity; and K is permeability in m^2. Note that K in the formulation is different from the permeability coefficient k (m/s), which is used in soil mechanics as follows:

$$K = k\frac{\eta}{\rho_f g} \tag{9}$$

The boundary conditions described previously can be written as

$$z = 0: \qquad u = U_0 e^{i\omega t},\ w = 0 \tag{10}$$

$$z = L: \qquad p_f = 0,\ \sigma = 0 \tag{11}$$

With the aid of a de-coupled procedure (Yang & Sato 2000b), equations (7) and (8) can be finally solved by enforcing the boundary conditions as

$$u(z, t) = \frac{U_0}{\delta_1 - \delta_2}\left\{\delta_1\frac{\cos[k_1(L - z)]}{\cos(k_1 L)} - \delta_2\frac{\cos[k_2(L - z)]}{\cos(k_2 L)}\right\}e^{i\omega t} \tag{12}$$

$$w(z, t) = \frac{U_0}{\delta_1 - \delta_2}\left\{\frac{\cos[k_1(L - z)]}{\cos(k_1 L)} - \frac{\cos[k_2(L - z)]}{\cos(k_2 L)}\right\}e^{i\omega t} \tag{13}$$

where k_1 and k_2 are, respectively, two complex wave numbers for the two types of compression waves in porous soils, and δ_1 and δ_2 are two frequency-dependent quantities that satisfy the following equation (Yang & Sato 2000b):

$$[\rho_f(\lambda_c + 2\mu) - \rho\alpha M]\delta^2 + [(\lambda_c + 2\mu)(m - ib/\omega) - \rho M]\delta + [\alpha M(m - ib/\omega) - \rho_f M] = 0 \tag{14}$$

The total stress and pore pressure can then be derived from equations (3) and (4) as

$$\sigma(z, t) = \frac{U_0}{\delta_1 - \delta_2}\left\{\begin{array}{l}[(\lambda_c + 2\mu)\delta_1 + \alpha M]k_1\dfrac{\sin[k_1(L - z)]}{\cos(k_1 L)} \\ -[(\lambda_c + 2\mu)\delta_2 + \alpha M]k_2\dfrac{\sin[k_2(L - z)]}{\cos(k_2 L)}\end{array}\right\}e^{i\omega t} \tag{15}$$

$$p_f(z, t) = \frac{U_0}{\delta_1 - \delta_2}\left\{\begin{array}{l}(M + \alpha M\delta_2)k_2\dfrac{\sin[k_2(L - z)]}{\cos(k_2 L)} \\ -(M + \alpha M\delta_1)k_1\dfrac{\sin[k_1(L - z)]}{\cos(k_1 L)}\end{array}\right\}e^{i\omega t} \tag{16}$$

Based on these rigorous solutions, some fundamental features related to the responses of stress and pore pressure in the soil layer can be deduced. The interest of this study is focused on the motion amplification, which is readily defined herein as the ratio of the amplitude of solid displacement at any depth to that at the rigid base as

$$T(z, \omega) = \frac{1}{\delta_1 - \delta_2}\left\{\delta_1\frac{\cos[k_1(L - z)]}{\cos(k_1 L)} - \delta_2\frac{\cos[k_2(L - z)]}{\cos(k_2 L)}\right\} \tag{17}$$

By choosing $z = L$ the transfer/amplification function for the deposit is given as

$$T(L, \omega) = \frac{1}{\delta_1 - \delta_2}\left(\delta_1\frac{1}{\cos(k_1 L)} - \delta_2\frac{1}{\cos(k_2 L)}\right) \tag{18}$$

Apparently, the transfer function depends on the degree of saturation, soil properties (porosity, permeability, stiffness etc.) and the layer thickness, as well as the loading frequency through quantities of k_1, k_2, δ_1, δ_2 and L.

NUMERICAL EXAMPLES

In this section, numerical examples are given to illustrate the influence of saturation on the amplification in vertical motion. Gravelly soil, denoted as G50 in Kokusho & Yoshida (1997), is used in calculation. The physical properties of this soil are listed in Table 1. The bulk modulus of solid grains is assumed to be around $3{\cdot}6 \times 10^{10}$ N/m^2, and the Poisson's ratio for the skeleton is taken as a typical value of 0·3. The fluid viscosity and permeability are taken as 10^{-3} Pa s and 10^{-9} m^2. The shear wave velocity of the gravelly soil, V_s, may be determined by the following formula, obtained based on laboratory data (Kokusho & Yoshida, 1997):

$$V_s = \left\{120 + \left[420\frac{U_c}{U_c + 1} - 120\right]D_r\right\}\left(\frac{\sigma_v \sigma_h}{p_0^2}\right)^{0{\cdot}125} \qquad (19)$$

where σ_v and σ_h are vertical and horizontal stress, p_0 is reference pressure (98 kPa), U_c is the uniformity coefficient, and D_r is relative density. In the following computation three relative densities, 25%, 50% and 75%, are considered. The vertical stress is taken as an average value at the mid-height level of the soil layer, and the horizontal stress is calculated as $\sigma_h = K_0 \sigma_v$, where K_0 is the coefficient of earth pressure. As suggested by experimental data, an average value of $K_0 = 1/3$ is selected for the calculation. The thickness of the layer is assumed as 12 m and the absolute fluid pressure, p_a, is approximately taken as 150 kPa.

Table 1. Physical properties of the gravelly soil used in examples

Specific gravity of soil particles	2·668
Maximum void ratio	0·429
Minimum void ratio	0·240
Mean grain size, D_{50} (mm)	2·28
Uniformity coefficient	11·3

Figures 3(a) and (b) show the amplification at the layer surface as a function of frequency for several different degrees of saturation, for $D_r = 25\%$. A noticeable difference is found between the case of incomplete saturation and the case of full saturation. Even if the degree of saturation is only slightly below full saturation, the peak frequency is substantially shifted to the low-frequency end. This performance is reasonable, since air inclusion can lead to a dramatic decrease in P-wave velocity, as will be shown later. Similar performance is also observed for soils with $D_r = 50\%$ and $D_r = 75\%$, as shown in Figs 3(c) and (d).

The rate of the reduction in peak frequency due to decreasing saturation drops rapidly. For example, for $D_r = 25\%$, the reduction in fundamental frequency (the lowest peak frequency) is around 27 Hz when S_r changes from 100% to 99·9%, whereas it drops to only 1 Hz when S_r decreases from 99% to 98%. This feature is clearly depicted in Fig. 4(a), in which the fundamental frequency is presented as a function of saturation for the three relative densities considered. It is of interest to note that the variation of fundamental frequency with saturation is similar way to the variation of P-wave velocity variation with saturation, as shown in Fig. 4(b), implying a reasonable correlation between them.

Owing to the shift of frequency content, the transfer function for the case of full saturation may, at a certain frequency, yield

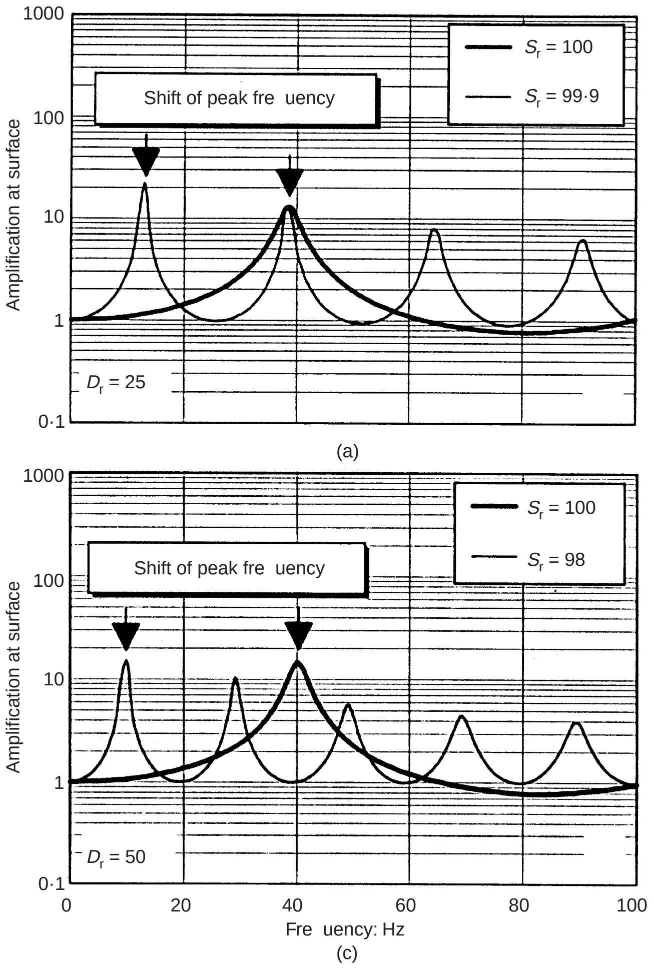

Fig. 3. Amplification at the surface as a function of frequency

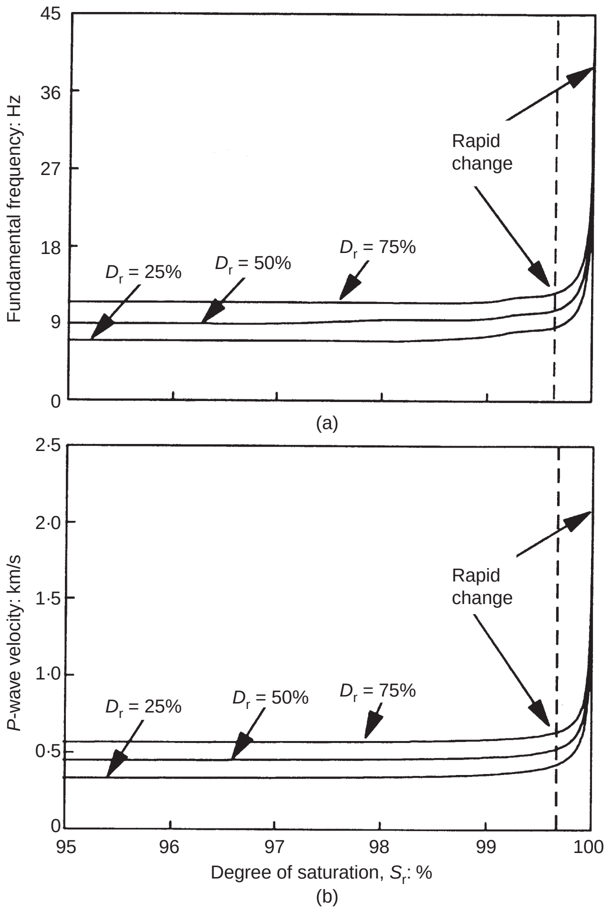

Fig. 4. Variation of fundamental frequency and *P*-wave velocity with saturation

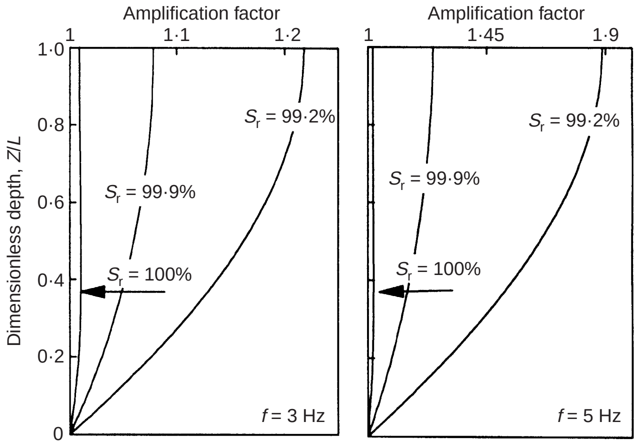

Fig. 6. Distribution of amplification factor with depth for two specified frequencies

the same amplification as that determined from the transfer function for the case of incomplete saturation, as seen in Fig. 3. In the case of earthquake loading, the frequencies of interest are usually not high. For example, the predominant frequency in vertical motion recorded at the array site is around 4–5 Hz (Yang & Sato, 2000a). For this reason, a better view of the amplification in a low-frequency range is given in Fig. 5 for $D_r = 25\%$. Now it is found that a saturated model will always underestimate the motion amplification. This performance is also exhibited in Fig. 6, which illustrates the distribution of motion amplification with depth for two specified frequencies of 3 and 5 Hz. Similar behaviour is also observed for $D_r = 50\%$ and $D_r = 75\%$. For the sake of conciseness those results are not shown here.

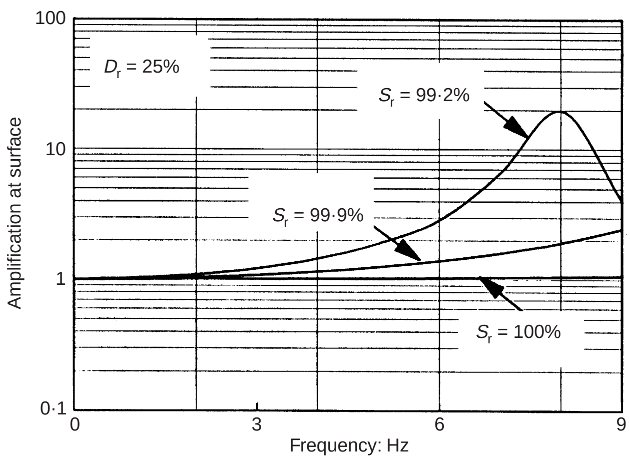

Fig. 5. Effects of saturation on motion amplification in the low-frequency range

In addition to the degree of saturation, soil properties and loading frequency, the amplification also depends on the thickness of the soil deposit. On the basis of the derived transfer function, the amplification at the surface is computed for several cases of thickness. As expected, the results show a reasonable feature that the fundamental frequency decreases with increasing layer thickness.

If one treats the transfer function for the case of incomplete saturation as the correct one, which reflects the true condition of the site, while the incorrect transfer function is the one presumably obtained without considering such soil condition, the results described above imply that the estimate of vertical motion amplification would be incorrect if the condition of incomplete saturation was ignored: that is, if it is assumed that the soil below the water table is fully saturated, as done in most site response analyses. This conclusion suggests the importance of a correct evaluation of saturation for soil deposits.

It is interesting to note that, in their recent work on site response analysis for the 1986 Lotung Earthquake, Li *et al.* (1998) pointed out that the calculated results at the surface 'do not match [the records] as well in the vertical direction, ... with *the amplification factors of the calculated motions in the vertical direction being noticeably lower than the recorded ones*', and they concluded that 'this observation suggests that the compressibility of voids used in the analyses was too low, which implies that the soils in the upper layers *might not be completely saturated*'. The present study seems able to clarify their inference to some extent.

CONCLUSIONS

An analysis has been presented of the saturation effect on the amplification of a soil layer to vertical earthquake excitation. The motion amplification was derived as a function of degree of saturation, soil properties, layer thickness and loading frequency. Numerical results indicate that, even if the degree of saturation is only slightly below full saturation, its impact on motion amplification is significant. An underestimation of vertical amplification would take place if the condition of incomplete saturation was ignored. The present study provides a physical insight into the field observation of large amplification in vertical motion at a reclaimed site during the Kobe earthquake. Further research toward a correct evaluation of *in situ* saturation of soil deposits and an appropriate incorporation of the saturation effect in site response analysis would be of value. An attempt has been made for the Kobe case history (Yang & Sato, 2000a).

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