State variables for silty sands: Global void ratio or skeleton void ratio?

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Abstract

While the global void ratio has long been used as a density index to characterize sand behavior, concern has been increasing about its applicability to silty sands (sand–fines mixtures), based on the proposition that the fines may fill in the void spaces formed by sand grains and make no contribution to the force transfer. The skeleton void ratio was proposed in the literature as an alternative index for mixed soils, based on the assumption that all fines act as voids. It was further modified into an equivalent skeleton void ratio by taking into consideration the fraction of fines that participates in the force transfer. This paper presents a study aimed at evaluating the three state variables as applied to sand–fines mixtures and especially to explore the rationale behind the concept of the skeleton void ratio. Based on a specifically designed experimental program, it is shown that contrasting conclusions can be drawn as to the role of fines in altering the shear behavior of clean sand when different density indices are used as the comparison basis. When comparisons are made at a constant (global) void ratio, the fines increase the degree of contractiveness, but when comparisons are made at a constant skeleton void ratio, an increase in dilativeness is seen. The equivalent skeleton void ratio does not fulfill the intent of providing a universal means for characterizing the stress–strain behavior of silty sands. This is due to the lack of mechanisms to account for the inter-granular contacts which are highly complex. The study suggests that compared with the skeleton void ratio and its modified form, the usual (global) void ratio remains a simple and useful state variable suitable for the framework of critical state soil mechanics and for geotechnical applications.

1. Introduction

The global void ratio, defined as the volume of voids divided by the volume of solids, has long been used in soil mechanics as a density parameter to characterize soil behavior. Fig. 1 schematically shows three distinct responses of saturated sand to undrained shearing, characterized by the post-consolidation void ratio. Under otherwise similar conditions, the dense specimen exhibits a strain-hardening response, whereas the loose specimen exhibits a highly contractive response with a marked build-up of pore pressures leading to the failure known as static or flow liquefaction. At an intermediate density, the sand contracts in the initial stage of shear and then dilates continuously to large strains, with the phase transformation state marking the transition. Various aspects of the density-dependent stress–strain–strength behavior of sands (e.g., Castro and Poulos, 1977; Alarcon-Guzman et al., 1988; Ishihara, 1993; Yang and Li, 2004; and the references therein) have been characterized within the framework of critical state soil mechanics, which defines a
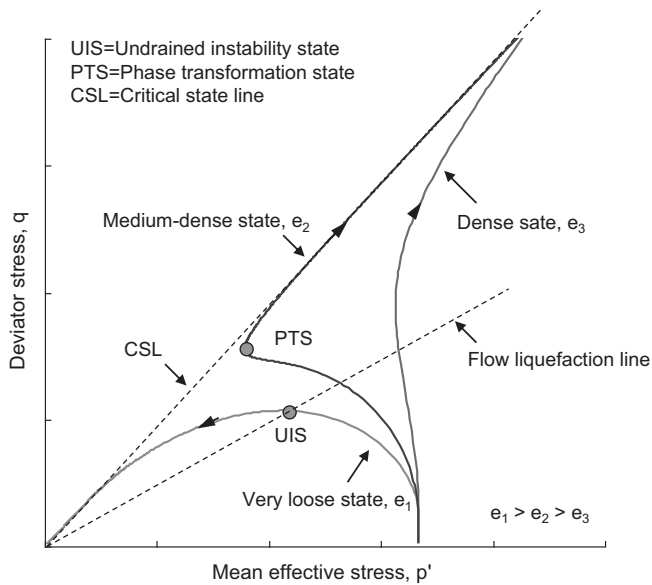

Fig. 1. Schematic illustration of typical undrained shear responses of sand at different void ratios.

unique critical state locus (CSL) in the void ratio–mean effective stress (i.e., e–p') plane such that the locus serves as a boundary separating the initial states of sand into contractive and dilative regions (Been and Jefferies, 1985; Wood, 1990; Verdugo and Ishihara, 1996).

When silt or clay fines are present in clean sand, the sand's behavior may be significantly altered. A number of experimental studies (e.g., Pitman et al., 1994; Lade and Yamamuro, 1997; Thevanayagam et al., 2002; Georgiannou, 2006; Murthy et al., 2007) have provided data showing the effect of fines in undrained loading conditions. Nevertheless, very diverse views exist on whether the effect of fines is negative or positive for the shear strength and liquefaction potential of sand (Yang and Wei, 2012). Concerns have arisen about the effectiveness of the usual void ratio in characterizing the behavior of such mixed soils. Based on the hypothesis that fines may roll into the voids formed by sand grains, and hence, make little contribution to the force transfer mechanism (e.g., Mitchell, 1976), an index known as the skeleton void ratio was used as an alternative to characterize the mixtures of sand and fines in several studies (Kuerbis et al., 1988; Georgiannou et al., 1990; Pitman et al., 1994; Thevanayagam, 1998; Chu and Leong, 2002). The skeleton void ratio (e_s) is related to the conventional void ratio (e) as follows:

$$e_s = \frac{e + FC}{1 - FC} \tag{1}$$

where FC denotes the percentage of fines content. For clean sand with a zero fines content, e_s is exactly the same as e. In deriving Eq. (1), the specific gravity of fines is assumed to be similar to that of sand grains.

Recognizing that not all fines would act as voids at a high fines content, the concept of the skeleton void ratio was further modified (Thevanayagam et al., 2002) to give an index referred

to hereafter as the equivalent skeleton void ratio

$$e_{se} = \frac{e + (1 - b)FC}{1 - (1 - b)FC} \tag{2}$$

where factor b, varying between 0 and 1, represents the fraction of fines that contributes to the force structure. Evidently, when b is zero, e_{se} reduces to e_s, meaning that the fines act as voids; when b is 1, e_{se} reduces to e, meaning that the fines act like the particles of the base sand. Note that when using Eq. (2), the fines content (FC) should be less than the threshold fines content (30–40% for most mixed soils), so that the mixed soil can be treated as being sand-dominated. Also, the fines should be non-plastic, so that the external forces can be assumed to be transmitted by direct inter-granular contacts without the chemical–physical effects of plasticity fines (Yang and Wei, 2012).

In recent years, interest has been growing in the use of the equivalent skeleton void ratio to characterize the behavior of sand–fines mixtures (e.g., Ni et al., 2004; 2006; Yang et al., 2006; Rahman et al., 2008; Rahman and Lo, 2012). The key step in doing that is the determination of factor b in Eq. (2). Most studies employed the best-fit approach to obtain the b value such that the critical state data of the base sand and its mixture with fines, when plotted in the e_{se}–p' plane, fall within a narrow band to give a single CSL. Fig. 2 illustrates the idea using the triaxial test data of Zlatovic and Ishihara (1995) on a clean sand mixed with non-plastic fines. As can be seen from Fig. 2(a), the CSL of the mixed soil in the e–p' plane tends to move downward as the quantity of fines increases. However, when these data are plotted in the e_{se}–p' plane (Fig. 2(b)), where e_{se} is calculated using $b = 0.25$ as given by Ni et al. (2004), they tend to fall in a narrow band for which a best-fit CSL can be derived.

While the idea appears to be attractive, it is worth noting that the position of the best-fit CSL is different from the position of the CSL of the base sand determined by using the critical state data on its own, as readily seen in Fig. 2(b). The CSL of the base sand is therefore no longer unique as it depends on the fines added; *obviously this violates the principle of the critical state approach that specifies the existence of a unique CSL for a given sand*, rendering the concept of the equivalent skeleton void ratio logically inconsistent with its premise.

Another confusing issue in the literature is the existence of multiple b values for a given dataset. For example, for the test results on an alluvium sand mixed with 9% non-plastic fines, Ni et al. (2004) selected $b = 0.7$ for characterizing the steady state or critical state strength of the mixed soil. For the same dataset, Rahman et al. (2008) predicted the value of b to be as low as 0.033 by using a semi-empirical formula that they had developed by analyzing several sets of published data. According to the definition given in Eq. (2), $b = 0.7$ means that 70% of the fines participate in the force transfer, whereas $b = 0.033$ means that less than 4% of the fines participate in the force transfer.

Evidently, if the concept of the equivalent skeleton void ratio is to become more widely accepted, research is needed
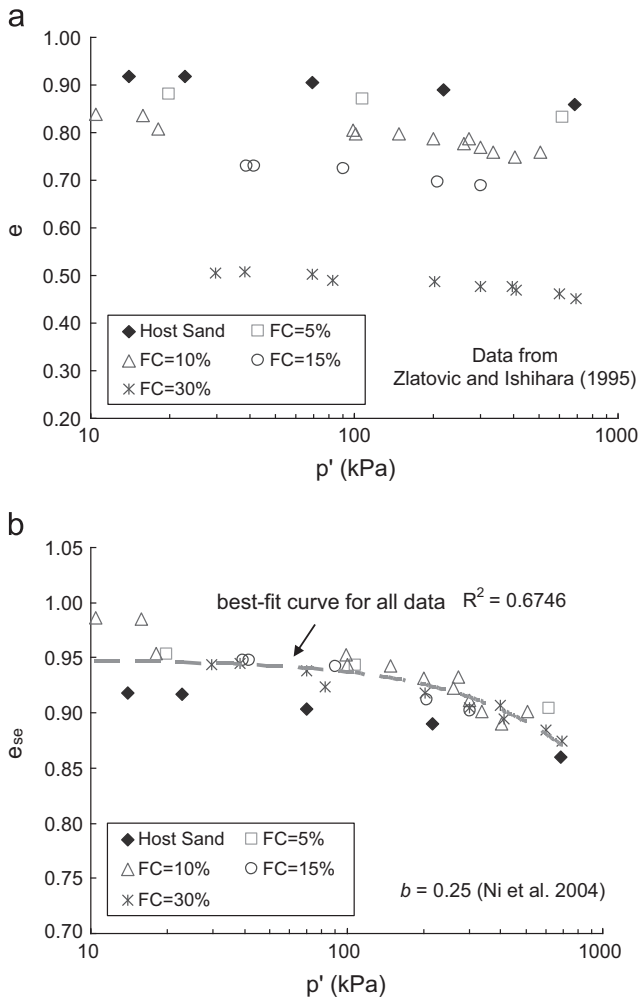
Fig. 2. Critical state data for a clean sand and its mixtures with fines in (a) the e–p' plane and (b) the e_{se}–p' plane.

Fig. 3. Particle size distribution curves and SEM images of test materials.

Table 1
Index properties of test materials.

	Leighton buzzard sand	Fujian sand	Toyoura sand	Crushed silica
G_s	2.65	2.65	2.65	2.68
D_{10} (μm)	634.9	282.0	166.0	25.5
D_{50} (μm)	877.1	397.0	216.0	53.9
D_{60} (μm)	937.7	432.0	231.0	55.7
C_u	1.477	1.532	1.392	2.182

to address the above issues and especially to explore the underlying rationale. The key hypothesis behind the concept is that if a mixed soil and its base sand are packed at the same value of e_{se} and loaded under the same conditions, they should behave in the same way. To examine this hypothesis, it is highly desirable to acquire experimental datasets for a range of sand–fines mixtures that allow for a systematic comparison of the stress–strain behavior of clean sands and their mixtures at similar values for e_{se}. These datasets can also serve as a useful reference for the development of advanced constitutive models for mixed soils. With this aim, a specifically designed experimental program has been carried out that covers a range of mixed soils in terms of size ratio, fines content and grain shape. In this paper, these extensive test series are examined and interpreted using the three different density parameters, and their feasibility to characterize the shear behavior of the mixed soils is carefully assessed.

2. Testing program

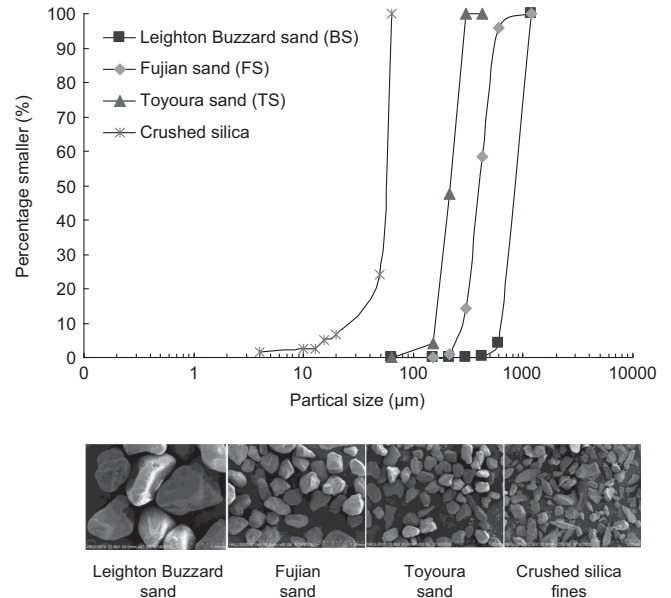One of the features of the testing program is the coverage of a range in sand–fines mixtures at different size ratios. Three clean quartz sands, Toyoura sand (TS), Fujian sand (FS) and Leighton Buzzard sand (BS), were used as the three base sands. Their grading curves and scanning electron microscopy (SEM) photos are shown in Fig. 3. The three sands have similar uniformity, but different mean sizes (Table 1). Non-plastic crushed silica was used as the fines to mix with the base sands, giving three types of sand–fines mixtures. For each type of mixture, the percentage of fines varied from 5% to 15% to take into account the influence of the fines content. In the following discussion, shorthand notations are used for the mixed soils; for example, TSS(5) stands for Toyoura sand mixed with 5% silica fines and FSS(10) stands for Fujian sand mixed with 10% silica fines.

The other key feature of the testing program is the coverage of a broad range in packing densities, and thus, a range in shear behavior from highly contractive to dilative. The moist tamping method with the under-compaction technique (Ladd, 1974; Ishihara, 1993) was used to prepare the samples. Each reconstituted specimen was measured to be 71.1 mm in diameter and 142.2 mm in height, and was saturated in two stages – initially by flushing the specimen with carbon dioxide and de-aired water and then by applying back pressure. The range in back pressure was between 240 and 340 kPa. After saturation, the specimen was isotropically consolidated to the targeted confining stress. Undrained shearing was then

undertaken with a strain rate of 0.5%/min. As the test materials involved fine to medium sands, the effect of membrane penetration was found to be insignificant, and thus, was not taken into account.

Care should be taken in the determination of the void ratio prior to shearing for sand–fines mixtures. Two methods were used in the study to determine the global void ratio for the soil specimens. The first method was based on the measurements of the initial void ratio during preparation and the volumetric strain that the specimen underwent during consolidation. The second method was similar in principle to that of Verdugo and Ishihara (1996) and was based on the measurement of the water content at the end of the test. With due care and diligence, the two methods can give a reasonably good

agreement (Yang and Wei, 2012). For mixtures with a high fines content and high compressibility, greater care should be taken and the second method is recommended.

3. Stress–strain behavior and stress path

3.1. Shear behavior compared at a similar void ratio

Fig. 4(a) shows the stress paths and stress–strain curves of two TSS specimens along with those of the base sand (TS) at a similar post-consolidation void ratio (e). All specimens were sheared at the mean effective stress of $p' = 500$ kPa. The clean sand specimen exhibited a highly dilative, strain-hardening response, whereas mixed soil specimen TSS(5) displayed a
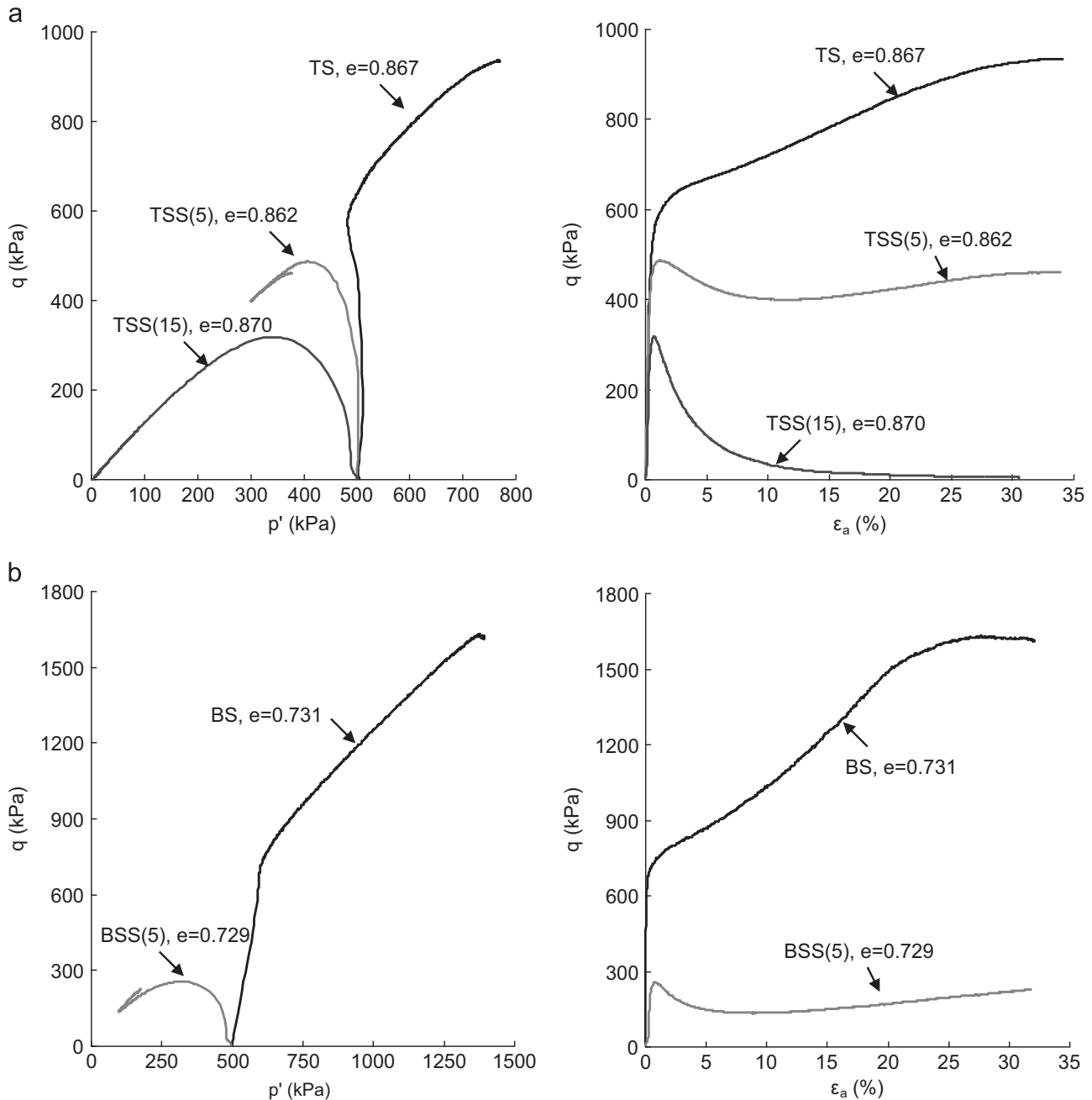

Fig. 4. Undrained shear responses of clean sand and its mixtures with crushed silica fines at a similar void ratio: (a) TS and TSS and (b) BS and BSS.

contractive response with a significant reduction in strength. When the percentage of fines was increased to 10%, the reduction in strength became more remarkable. And, at a fines content of 15%, the mixed soil specimen underwent complete liquefaction with zero residual strength at large strains.

Similar observations were also obtained for FSS and BSS specimens. Given the limited space, only one pair of specimens – BS and BSS(5), sheared from a similar initial state ($e = \sim 0.730$ and $p' = 500$ kPa) – are compared in Fig. 4(b).

In addition, by comparing Fig. 4(a) and (b), the increase in contractiveness, due to the presence of fines, appears to be more significant for mixture BSS than for mixture TSS. This difference is considered to be associated mainly with the difference in the size disparity between the coarse and fine particles of the two mixtures. Compared with TSS, BSS has a markedly large-size disparity, which may facilitate the movement of fines into the void spaces, thus leading to the soil structure being more unstable. Further discussion on this point will be given in a later section.

3.2. Shear behavior compared at a similar skeleton void ratio

The concept of the skeleton void ratio (e_s) assumes that a mixed soil with a small amount of fines should behave similarly to its base sand if both are packed at a similar value for e_s. Fig. 5 presents experimental data for examining whether or not this assumption holds true. For the two plots in Fig. 5(a), the stress paths and stress–strain curves for clean sand specimen TS ($e_s = 0.950$) and a specimen of the same sand mixed with 5% fines ($e_s = 0.945$) are compared. Both specimens were sheared at the same confining stress (500 kPa). Although the fines content was controlled to be low, the two specimens behaved in distinctly different ways: the clean sand specimen underwent complete liquefaction, whereas the mixed soil specimen displayed a strong dilative response achieving high strength at large strains. Similar observations were also made for FSS and BSS specimens, and an example is given in Fig. 5 (b).

The test results in Fig. 5 indicate that if the skeleton void ratio is used as the state variable for comparison, the presence of fines enhances the dilatancy of the sand and contributes to the strength and liquefaction resistance. This is in contrast to the conclusion derived when using the global void ratio as the comparison basis. Indeed, several studies in the literature have concluded that fines have a beneficial effect. However, such a conclusion is misleading as it is established by using the skeleton void ratio as the comparison basis, for which the key underlying assumption is that the fines act as voids and *make no contribution* to the force structure. This *logical inconsistency* suggests that the skeleton void ratio is not a rational state variable for characterizing mixed soils.

3.3. Shear behavior compared at a similar equivalent skeleton void ratio

The key step to using the equivalent skeleton void ratio (e_{se}) is to determine factor b. As discussed before, the current

approach, namely, best-fitting the critical state data for a mixed soil and its base sand with a single critical state locus (CSL) in the e_{se}–p' plane, is flawed in that the CSL so determined differs from the CSL of the base sand itself. In order to resolve this problem, it is proposed that the CSL of the base sand be fixed as the target in the e_{se}–p' plane and that the critical state data of the mixtures at different percentages of fines be fitted to this target through an optimum b value.

To elaborate the idea, firstly, the critical state data for the three types of mixtures were determined with reasonable diligence and care, as shown in the e–p' plane (Fig. 6). For most of the tests in this study, the developed axial strain level was over 30% and the rate of variation in deviatoric stress at that strain level was very small. From any practical point of view, such a state is considered close enough to the critical state (Verdugo and Ishihara, 1996). It is clear from Fig. 6 that, for each type of mixture, the CSL moves downward as the percentage of fines increases. This observation is consistent with that reported in the literature on several different mixed soils (e.g., Zlatovic and Ishihara, 1995; Thevanayagam et al., 2002; Rahman et al., 2008).

Moreover, it is noted that the CSL on the semi-log form is not a straight line, as is usually assumed, but rather a curved line. This curvature should not be attributed to particle breakage, because the stresses involved in the experiments were well below the stress level that may cause particle breakage of quartz sands (Verdugo and Ishihara, 1996; Ghafghazi et al., 2014). To give a better representation of the data, the CSL is described here by a power function (Li and Wang, 1998; Yang and Li, 2004)

$$e_c = e_\Gamma - \lambda_c \left(\frac{p_c}{p_a} \right)^\xi \tag{3}$$

where p_a is the atmospheric pressure taken as 101 kPa, and e_Γ, λ_c, and ξ are fitting parameters. Table 2 gives the fitting results for the three base sands tested. While some scatter exists in the critical state data, it mainly reflects the inherent variability in the material and the sensitivity of the critical state to the variation in void ratio; the fitting parameters or the location of the CSL is found to be insensitive to the scatter.

Next, mixed soil specimen TSS(5) was taken as an example. By assigning an initial value to b, the critical state data of TSS (5) were modified in terms of e_{se}, and the root mean square deviation (RMSD) of these data from the CSL of the base sand were then calculated. By varying the b value from 0 to 1 with an interval of 0.05, the variation in the RMSD with the b value could accordingly be determined, as shown in Fig. 7. It is clear that the optimum b value, to be chosen for this mixed soil, is that which corresponds to the lowest RMSD. Following this procedure, the optimum b values for all the mixed soils tested in this study were determined, as summarized in Table 3.

In the literature, a statistical criterion was often used to determine factor b such that the single CSL fitting all the critical state data (for both the base sand and its mixture) in the e_{se}–p' plane had an RMSD value of less than 0.043 (Yang et al., 2006; Rahman et al., 2008). If this benchmark value is
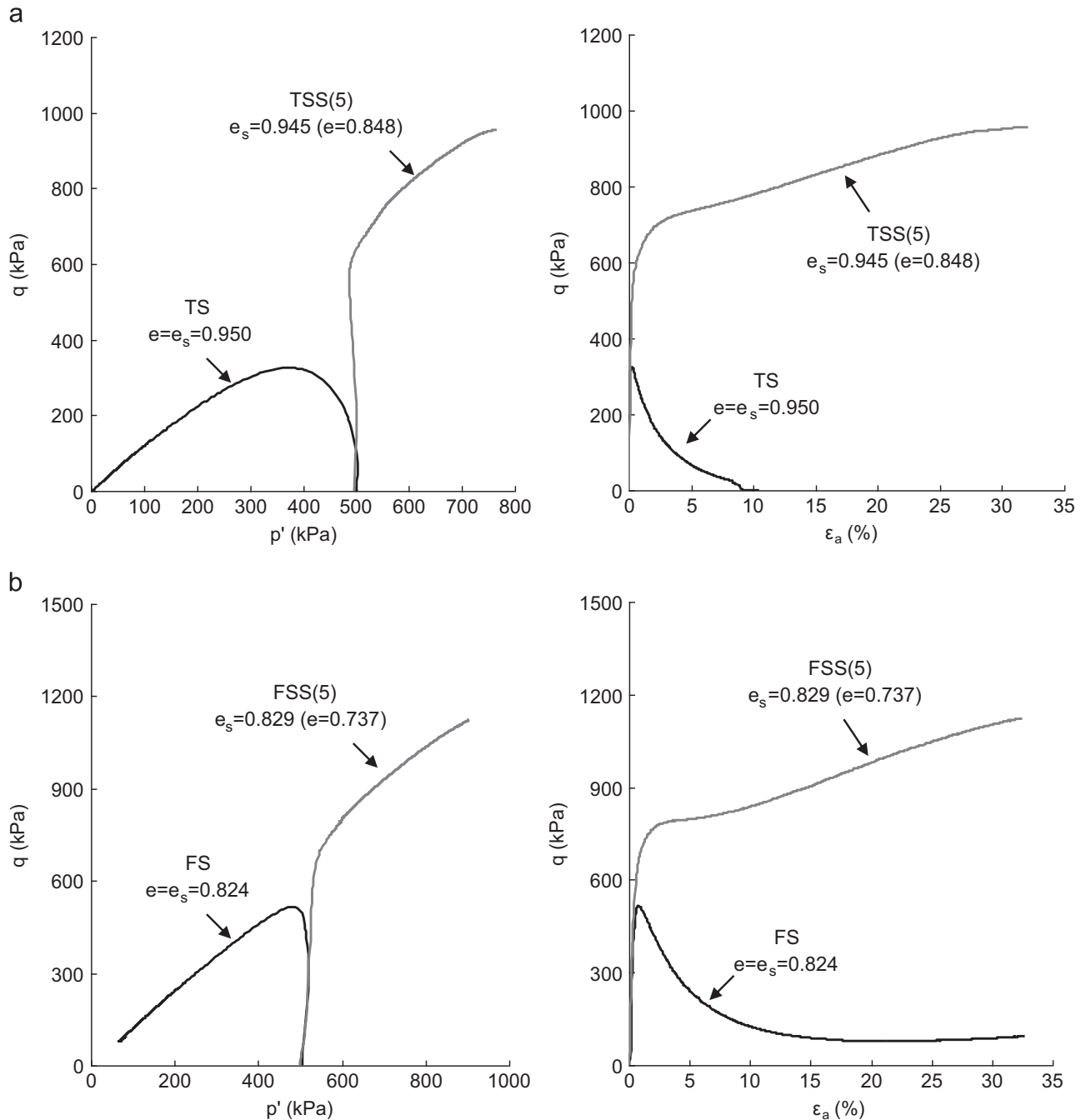
Fig. 5. Undrained shear responses of clean sand and its mixtures with crushed silica fines at a similar skeleton void ratio: (a) TS and TSS and (b) FS and FSS.

adopted here, as represented by the broken line in Fig. 7, then a range in b values meeting the criterion can be found. This might be a reason for the multiple b values reported in the literature for a given dataset.

Having determined the values for the factor b of all the three types of mixtures, the critical state data were then plotted in the e_{se}–p' plane (Fig. 8). As expected, the critical state data for each type of mixture fell in the vicinity of the CSL for the corresponding base sand.

It should be mentioned that factor b is treated here as a fines content-dependent quantity. This is considered more physically reasonable (Rahman et al., 2008) than assuming factor b to be independent of the fines content (e.g., Thevanayagam et al., 2002; Rees, 2010). Additionally, as can be seen in Fig. 8, this treatment results in the CSL being a much better representation of the critical state data.

The answer to the question of whether or not a mixed soil will behave in the same manner as its base sand, if they are packed at a similar e_{se}, is embodied in the plots in Fig. 9. In Fig. 9(a), the stress–strain response and the stress path of mixed soil specimen TSS(10) at $e_{se}=0.901$ are compared with those of clean sand specimen TS at $e_{se}=0.904$. Both specimens were sheared at $p'=500$ kPa. The overall behavior of the clean sand was more dilatant than the mixed soil. Although
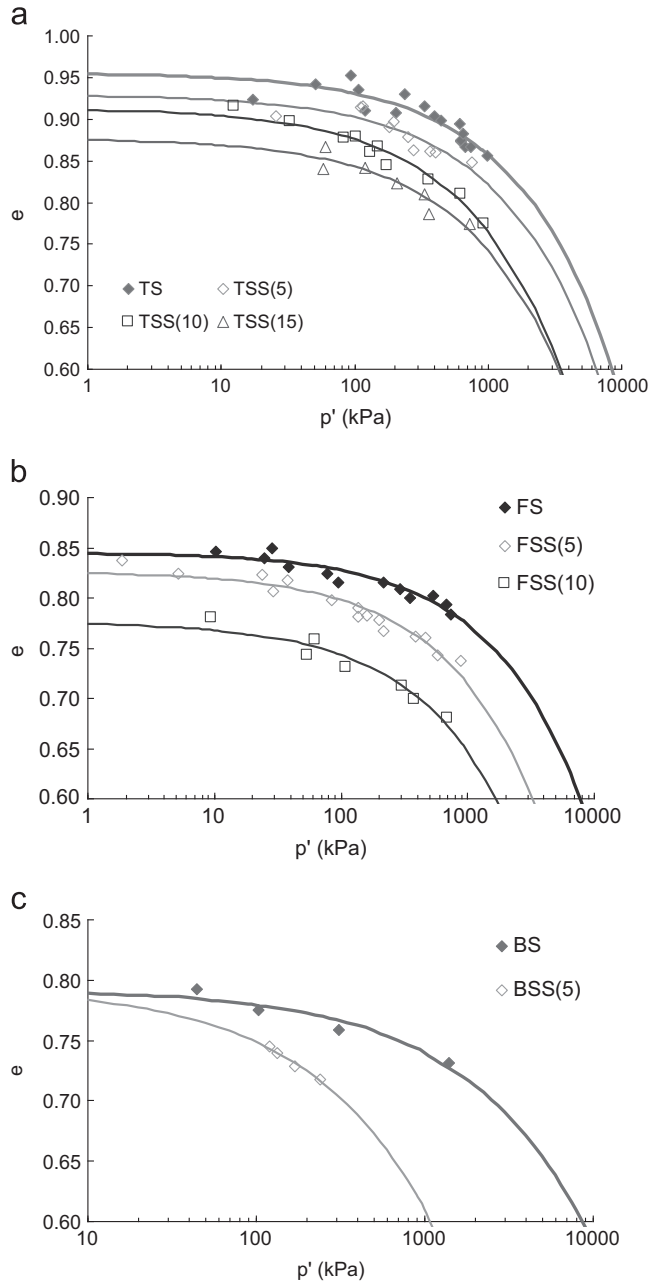
Fig. 6. Critical state loci of clean sand and its mixtures with crushed silica fines in the *e–p′* plane: (a) TS and TSS; (b) FS and FSS; and (c) BS and BSS.

Fig. 7. Example for determining the factor *b* using test data for TSS(5).

Table 3
Values of factor *b* for mixed soils tested.

Mixed soil	FC (%)	*b*	RMSD
TSS(5)	5	0.70	0.011
TSS(10)	10	0.70	0.017
TSS(15)	15	0.65	0.016
FSS(5)	5	0.60	0.011
FSS(10)	10	0.50	0.014
BSS(5)	5	0.45	0.008

Note: RMSD=Root of mean standard deviation.

gained the strength after the quasi-steady state; the mixed soil specimen, however, underwent a limited flow failure with a marked reduction in strength.

Similar observations were also obtained for the FS/FSS and BS/BSS specimens. Fig. 9(b) shows the direct comparisons of the stress–strain responses and the stress paths for clean sand specimen BS at $e=e_{se}=0.775$ and mixed soil specimen BSS (5) at a similar e_{se} (0.778). While both exhibited a contractive response in the initial stage of shearing and a similar dilative response after the quasi-steady state, they differed on two aspects: (a) the clean sand specimen achieved a significantly higher peak strength than the mixed soil specimen; and (b) the mixed soil specimen appeared to be more susceptible to the onset of flow liquefaction.

4. Undrained strength and onset of liquefaction

The undrained peak strength and the susceptibility to flow failure are two important considerations in geotechnical applications involving sandy soils. An effort is made here, therefore, to examine whether or not the skeleton void ratio or its modified form, as compared with the usual global void ratio, can provide a consistent means to characterize these two properties.

4.1. Undrained peak strength

The deviatoric stress at the peak state (also known as the undrained instability state in the literature) is linked to the

Table 2
Fitting parameters for critical state loci for clean sands.

Sand	e_Γ	λ_c	ξ	R^2
TS	0.955	0.025	0.6	0.83
FS	0.845	0.018	0.6	0.89
BS	0.800	0.015	0.6	0.98

Note: R^2=Coefficient of determination for CSL in the *e–p′* plane.

both specimens had similar peak strength, they showed significantly different post-peak responses: the clean sand specimen underwent a slight drop in strength and then re-
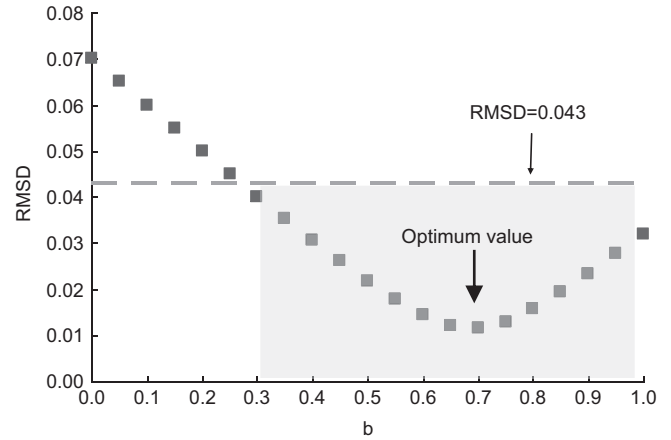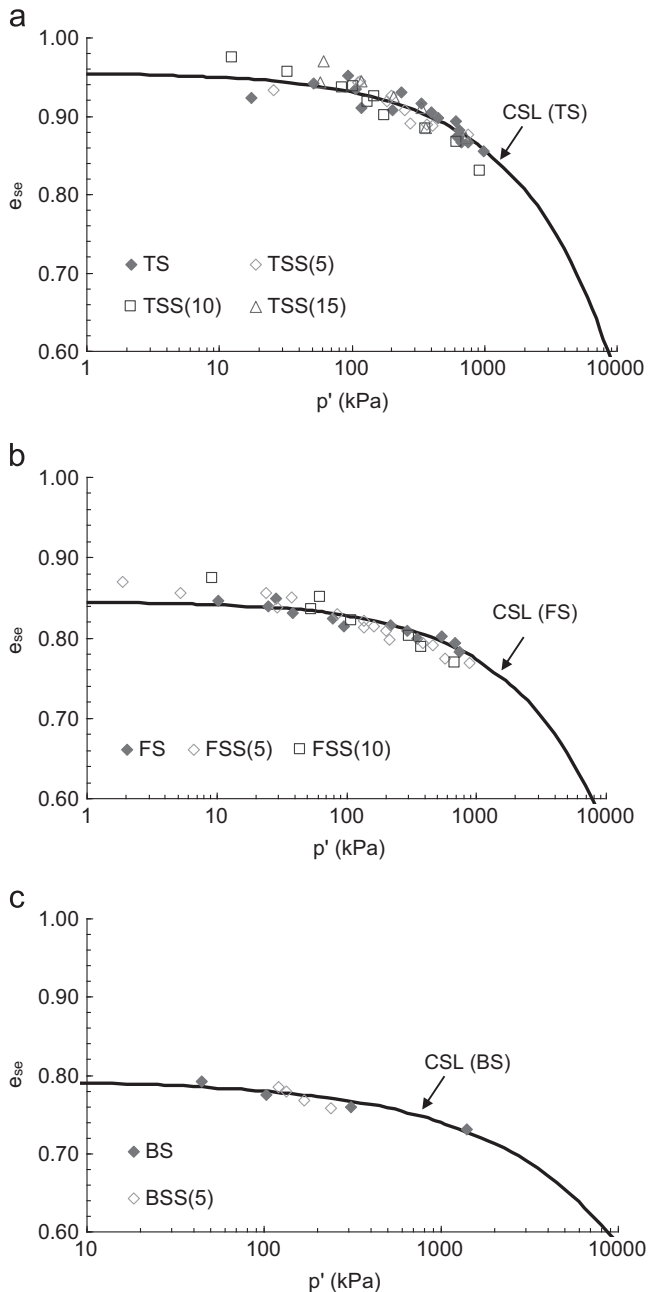
Fig. 8. Critical state data of clean sand and its mixtures with crushed silica fines in the e_{se}–p' plane: (a) TS and TSS; (b) FS and FSS; and (c) BS and BSS.

undrained peak strength. For the TSS and TS specimens, the values for this quantity (denoted as q_{UIS}) were derived as functions of e, e_s and e_{se}, respectively, as shown in the three plots in Fig. 10. Similar plots were also created for the FSS/FS and BSS/BS specimens (but given the limited space, they are not shown here). Several key observations can be made from these plots:

(a) When the global void ratio (e) is used as the comparison basis, the q_{UIS}–e curve tends to shift to the left as the amount of fines increases. This means that the effect of the fines is a decrease in peak strength at a given void ratio.

(b) When the skeleton void ratio (e_s) is used as the comparison basis, the q_{UIS}–e_s curve tends to shift to the right as the amount of fines increases. This means that the effect of the fines is an increase in peak strength at a constant skeleton void ratio. As discussed earlier, this view is misleading because of the logical inconsistency.

(c) When the equivalent skeleton void ratio (e_{se}) is used for the interpretation, data points for the mixed soils and the corresponding base sand tend to fall in a narrow band. Nevertheless, the mixed soils and the base sand do not simply follow a single relationship, implying that e_{se} cannot serve as a consistent means to characterize the undrained peak strength.

4.2. Onset of flow liquefaction

The deviatoric stress ratio at the instability state, defined as $(q/p')_{UIS}$, characterizes the onset of flow liquefaction (Vaid and Chern, 1985; Yang, 2002). The values for this stress ratio were determined for the BS and BSS specimens and are shown as a function of e, e_s and e_{se}, respectively (Fig. 11(a–c)). Similar plots for the TS/TSS and FS/FSS specimens were also obtained. The key observations for these plots can be summarized as follows:

(a) When the usual void ratio (e) is used to interpret the test data, the $(q/p')_{UIS}$–e curve tends to shift to the left as the amount of fines increases. This means flow liquefaction can be triggered more easily for mixtures with a higher fines content.

(b) When the skeleton void ratio (e_s) is used as the comparison basis, an opposite view is obtained, namely, that the clean sand tends to be more prone to the onset of flow liquefaction than the mixed soil. Again, this view is misleading because it is logically inconsistent with the premise of the skeleton void ratio.

(c) If the equivalent skeleton void ratio (e_{se}) is adopted as the comparison basis, the mixed soil appears to be more prone to the onset of flow liquefaction than the clean sand. However, the test data for the TS/TSS specimens indicate that the flow liquefaction can be more easily triggered in the clean sand than in the mixed soil at a similar e_{se} (Fig. 11(d)). This suggests that the equivalent skeleton void ratio fails to serve as a universal density parameter.

5. Significant role of grain shape

So far, three types of mixtures (TSS, FSS and BSS) have been examined. The fines used to form the mixtures were angular-shaped crushed silica. Given the important role of particle shape in altering the overall behavior of mixed soils (Yang and Wei, 2012; Wei and Yang, 2014), the applicability of the equivalent skeleton void ratio to mixed soils containing fines of distinct shapes was examined. Glass beads with a similar gradation and mean size to the crushed silica were used
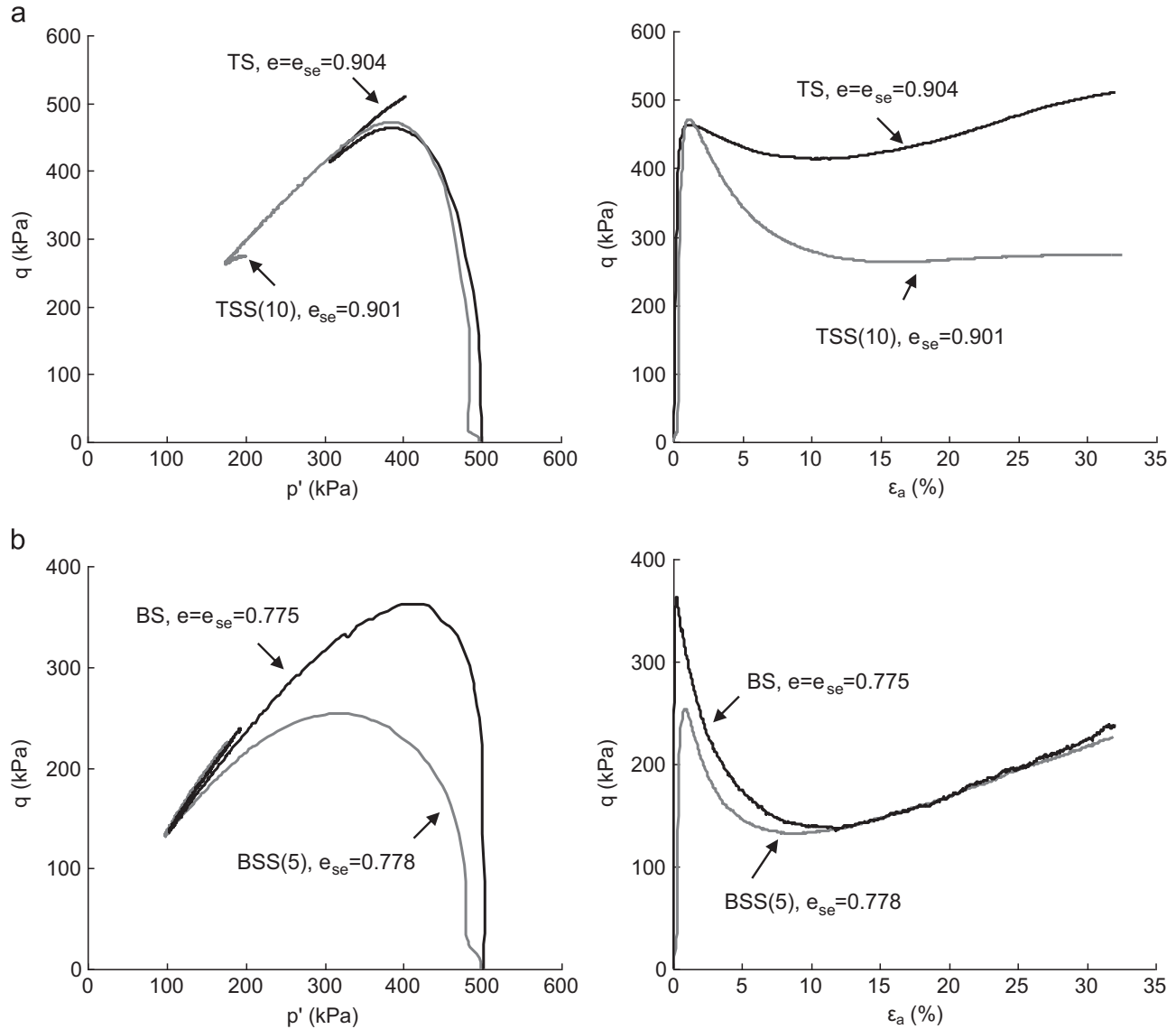
Fig. 9. Undrained shear responses of clean sand and its mixtures with crushed silica fines at a similar equivalent skeleton void ratio: (a) TS and TSS and (b) BS and BSS.

as an additional type of fines to mix with clean sand TS and FS. The fines content was set at 5%, and the formed mixtures are denoted here as TG(5) and FG(5), respectively. A series of tests were conducted for each type of mixture and the values for factor b were determined to be 0.8 for TG(5) and 0.6 for FG(5).

Fig. 12(a) compares the stress–strain behavior of one pair of mixed soil specimens, TG(5) and TSS(5), sheared at a similar e_{se}. Fig. 12(b) compares the stress–strain behavior of another pair of specimens, FG(5) and FSS(5). The effective confining stress for the four tests was 500 kPa. If the concept of the equivalent skeleton void ratio holds true, then the two specimens in each pair should have similar stress–strain behavior – as shown in Fig., 12(a and b), this is clearly not the case. For TSS(5) and TG(5), while both exhibit similar peak strength at small strains, their strengths at large strains are substantially different, giving different degrees of brittleness. For the pair of

specimens FSS(5) and FG(5), the one with glass beads as fines shows a markedly lower peak strength than the one with crushed silica as fines, but it exhibits a much stronger dilatancy after the occurrence of the quasi-steady state.

The discrepancies observed on the overall macroscale behavior at similar e_s or e_{se} are not considered unreasonable, and they are attributed to the highly complex particulate nature of the test materials. Further discussion on this aspect is given below.

6. Micromechanics-based considerations

The concept of the skeleton void ratio (e_s) assumes that all fines reside in the void spaces formed by the coarse grains. If this is true, then the loosest packing of a mixed soil should have $(e_s)_{max}$ which is close to the maximum void ratio (e_{max}) of the base sand. To examine this inference, the maximum
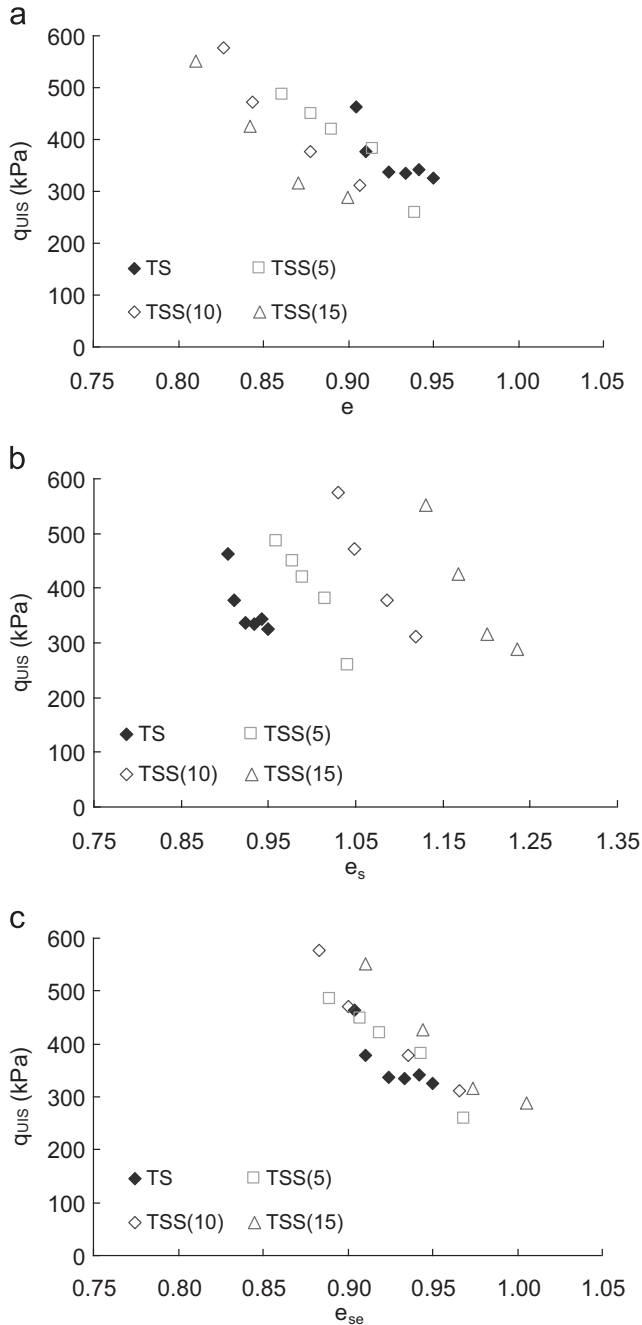
Fig. 10. Deviatoric stress at instability as a function of three different density parameters (data for TS/TSS specimens).

skeleton void ratio $(e_s)_{max}$ was calculated for each mixture according to its e_{max} that was determined by following the procedure specified in BS 1377: Part 4. Fig. 13 shows $(e_s)_{max}$ as a function of the fines content. For each type of mixture, the value of $(e_s)_{max}$ increases with an increasing fines content and it becomes substantially larger than the maximum void ratio (e_{max}) of the base sand when the fines content goes beyond 10%. This finding suggests that rather than all the fines rolling into the void spaces, some reside between the coarse grains and enlarge the soil skeleton. In this context, a larger size

disparity ratio is expected to increase the efficiency of the fines rolling into the voids, and thus, to give a smaller deviation of $(e_s)_{max}$ from e_{max}.

To verify the above hypothesis, the size disparity ratio – defined as $\chi = D_{10\_sand}/D_{50\_fine}$, where D_{10_sand} is the largest particle size in the smallest 10% of the sand particles and D_{50_fine} is the mean size of the fines – is calculated for all types of mixtures, as summarized in Table 4. Parameter χ is considered as an appropriate indicator of the mean size of voids (Aberg, 1992; Ni et al., 2004; Rahman et al., 2008). Interestingly, it is found that amongst BSS, FSS and TSS, the deviation between $(e_s)_{max}$ and e_{max} is indeed the smallest for BSS, which has the largest χ (11.78), whereas the deviation is the largest for TSS, which has the smallest χ (3.08).

In addition, given a similar size ratio and a similar percentage of fines, fine particles that are spherical and rounded are expected to be able to roll into the voids more efficiently than angular fines, thus leading to a lower deviation in the maximum void ratio. Strikingly, the $(e_s)_{max}$ of BG is indeed the closest to the e_{max} of its base sand (Fig. 13(c)) as compared with the other two types of mixtures.

The mechanisms discussed in the above paragraphs are considered to be responsible for the macroscale observation that the effect of fines in enhancing the contractiveness is more remarkable in BSS than in TSS (Fig. 4). From the micro-mechanics viewpoint, it is the soil structure that plays a fundamental role in the overall behavior of a mixed soil. The soil structure is associated with the distribution and type of inter-granular contacts, which are highly dependent on the characteristics of constituent particles, including their shape, size and mineralogy. Herein is an important implication: two mixed soil specimens, being packed at the same skeleton void ratio or equivalent skeleton void ratio, do not necessarily behave in the same way.

6.1. Types of inter-granular contacts in mixtures

It is hypothesized that three major types of inter-granular contacts exist in a sand–fines mixture: (1) the strong contact between coarse sand grains; (2) the sand–fine–sand contact, which is weaker than the first type; and (3) the sand–fine–fine–sand contact, which is the weakest. For clean sand or a mixture with a small amount of fines that is packed at a dense state, the first type of contact tends to prevail, making the formed soil structure the most stable. For a mixed soil with a high percentage of fines and packed at a loose state, the numbers of the second and third types of inter-granular contacts tend to increase, thus reducing the stability of the soil structure – this explains the observation that an increase in fines content increases the brittleness or collapsibility of the mixed soil. Furthermore, given the same type of inter-granular contact, if fine particles residing between the coarse grains are highly rounded, rather than angular, the soil structure tends to be more susceptible to volume changes, and subsequently, to collapse, as evidenced by the laboratory experiments of Yang and Wei (2012).

To verify the above hypothesis about the inter-granular contacts, an attempt was made to examine the soil structure
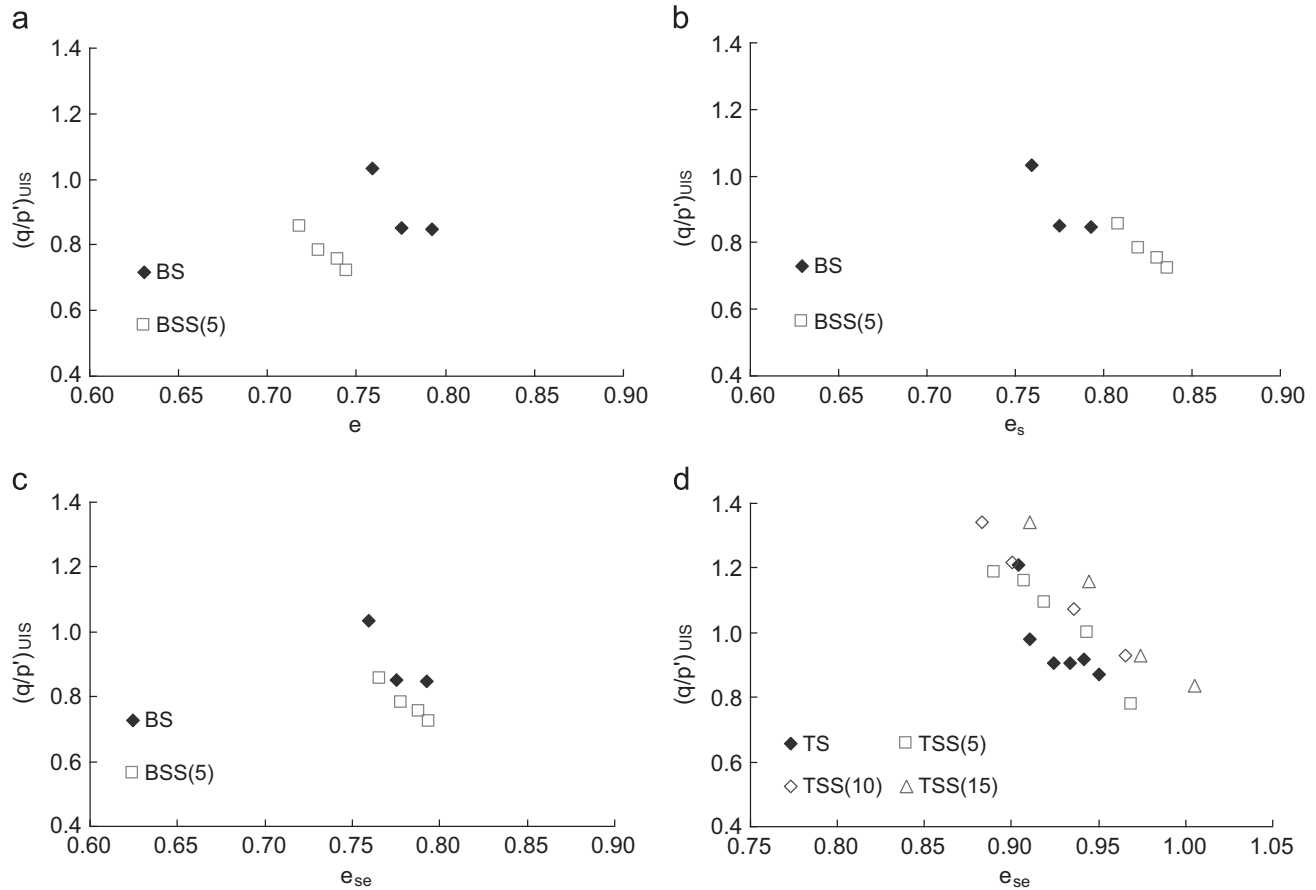
Fig. 11. Deviatoric stress ratio at instability as a function of three different density parameters: (a–c) data for BS/BSS specimens and (d) data for TSS/TSS specimens.

using the SEM technique in conjunction with the epoxy resin impregnation method (Jang et al., 1999; Yang et al., 2008). Fig. 14 shows an SEM photograph of a thin section taken from a specimen of Fujian sand containing 20% crushed silica fines, from which the three types of inter-grain contacts, discussed above, can clearly be identified. Of course, it would be of great value if a quantitative three-dimensional characterization could be made of the inter-granular contacts in a mixed soil specimen during the loading process; but the task is extremely difficult, if not impossible, in a physical experiment. Nevertheless, the task can be achieved by means of the discrete element modeling, and an attempt was presented in Luo and Yang (2013) for which several interesting results were given with special reference to the physical meaning of the equivalent skeleton void ratio.

7. Summary and conclusions

This paper has presented a study that aimed at evaluating the three different state variables for characterizing the shear behavior of sand–fines mixtures and to explore the rationale behind the concept of the skeleton void ratio. Systematic datasets have been created for a range in mixed soils, in terms of size disparity, fines content and grain shape, and these datasets have allowed for comparisons of the stress–strain responses of the mixed soils and their base sand at similar values of void ratio (e), skeleton void ratio (e_s) and equivalent skeleton void ratio (e_{se}). The main findings and results of the study are summarized as follows.

(a) The current approach to determining factor b – the center of the concept of the equivalent skeleton void ratio – is flawed in that the best-fit CSL becomes dependent on the fines added, and thus, deviates from the CSL of the base sand itself. This deviation violates the principle of the critical state approach that specifies a unique CSL for a given sand, rendering the concept of the equivalent skeleton void ratio logically inconsistent with its premise.

(b) When different density parameters are used as the comparison basis for a given dataset, diverse views can be derived on the effect of fines. Using the global void ratio (e), the effect of fines is found to be an increase in the degree of contractiveness, which is consistent with the observation that the addition of fines causes a downward movement of the CSL in the e–p' plane. This consistency suggests that the usual void ratio remains a useful state variable for characterizing the behavior of sand–fines mixtures in the critical state framework.
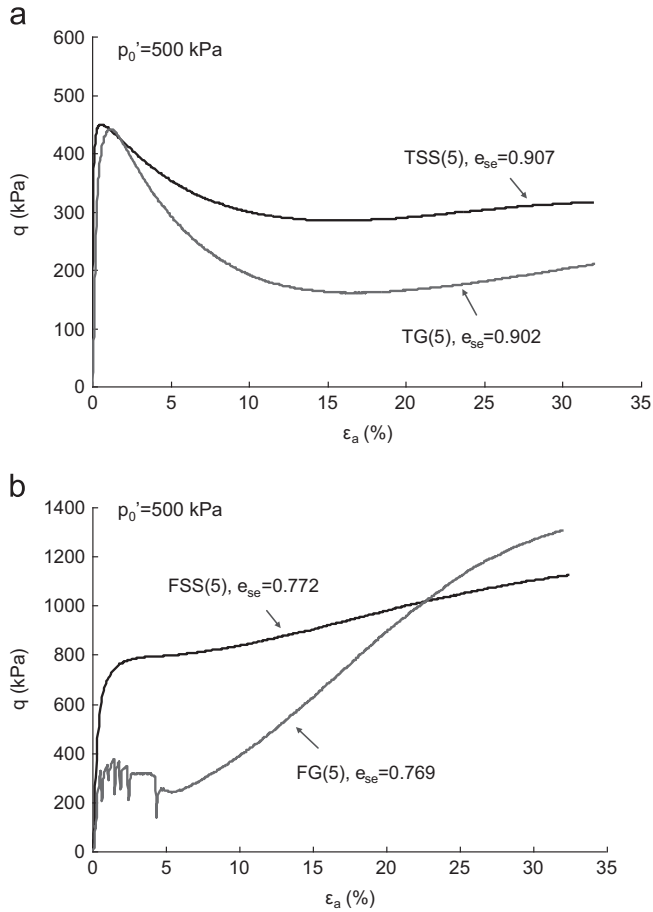
Fig. 12. Undrained shear responses of mixed soils with different types of fines compared at a similar equivalent skeleton void ratio: (a) TS and TG(5) and (b) FS and FG(5).

(c) When comparisons are made at a similar skeleton void ratio (e_s), the effect of fines is an increase in shear resistance accompanied by enhanced dilatancy. Caution should be taken with this view because it is logically inconsistent with the assumption underlying the concept of the skeleton void ratio that fines make no contribution to the force transfer.

(d) When comparisons are made at a similar equivalent skeleton void ratio (e_{se}), the mixed soils do not behave in the same way as their base sand in terms of the stress–strain relationship and the stress path, although in certain cases they may exhibit a similar peak strength or critical state strength. Caution should therefore be exercised in the use of the equivalent skeleton void ratio as a universal density parameter.

(e) The size disparity ratio and grain shape can impose a significant impact on the overall behavior of mixed soils. A large-size disparity ratio tends to promote the efficiency of the fines rolling into the voids and lead to the soil structure being metastable, and this tendency can be enhanced if the fines are more rounded. The inter-granular contacts or the associated soil structure plays a fundamental role in the
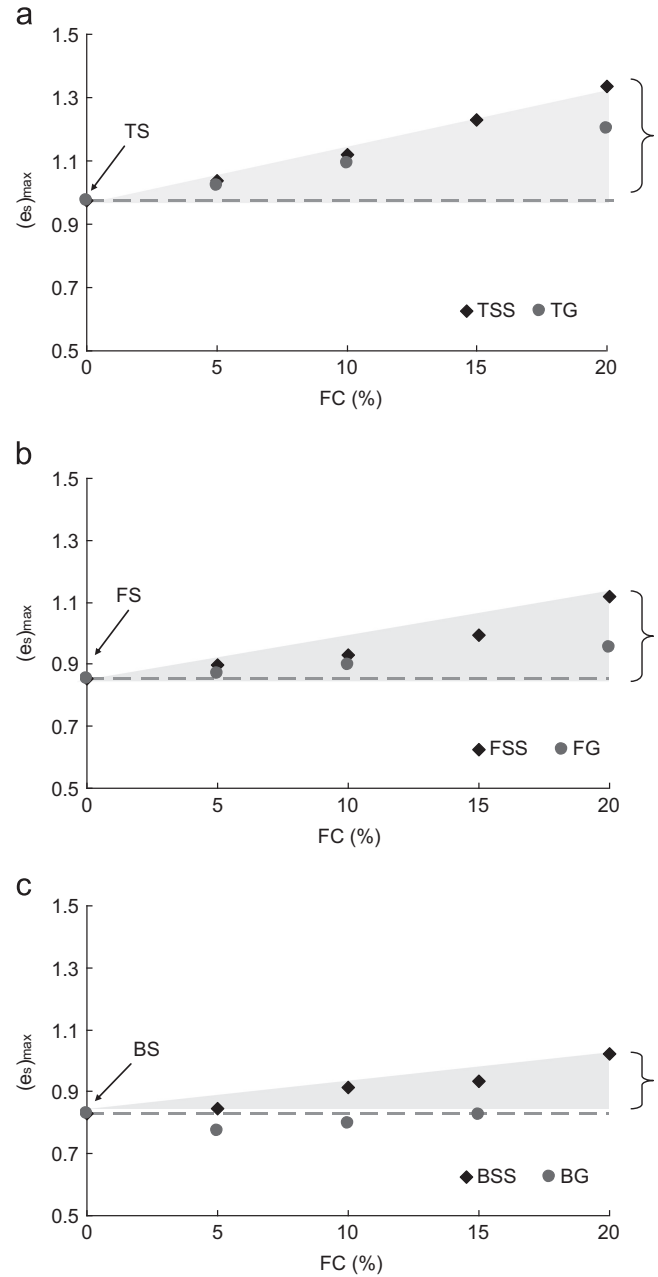

Fig. 13. Variation of maximum skeleton void ratio with fines content: (a) TSS and TG; (b) FSS and FG; and (c) BS and BG.

Table 4
Size disparity ratios of mixed soils tested.

	TSS	TG	FSS	FG	BSS	BG
$\chi = D_{10\_sand}/D_{50\_fine}$	3.08	3.81	5.23	6.47	11.78	14.56

D_{10_sand}: The largest particle size in the smallest 10% of sand particles.
D_{50_fine}: The mean size of fine particles.

macroscale behavior. The equivalent skeleton void ratio, as currently defined, lacks the mechanisms to account for these factors.
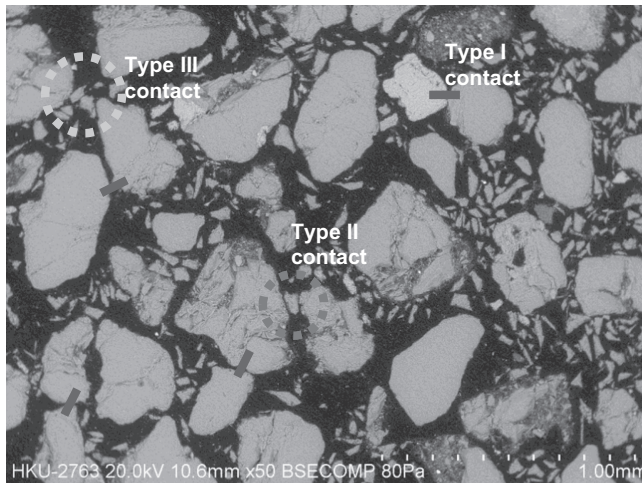
Fig. 14. SEM image analysis of soil structure: Fujian sand containing 20% silica fines.

Acknowledgments

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