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TECHNICAL NOTE

Liquefaction resistance of sand in relation to P-wave velocity

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KEYWORDS: dynamics; in-situ testing; laboratory tests; liquefaction; partial saturation; sands.

INTRODUCTION

The condition of partial saturation may produce two major impacts in earthquake geotechnical engineering. One is that partial saturation of subsoil can cause much greater amplification in vertical ground motion than for a fully saturated model, as demonstrated by an integrated study on a well-documented case from the Kobe earthquake (Yang & Sato, 2000, 2001). The other impact is related to the liquefaction potential of sandy soils. Laboratory tests (Sherif *et al.*, 1977; Chaney, 1978; Yoshimi *et al.*, 1989) have revealed that the liquefaction resistance of sands depends strongly upon the degree of saturation, which was expressed in terms of the pore pressure coefficient, B (Skempton, 1954; Lade & Hernandez, 1977). At a specified cyclic stress ratio, the number of cycles causing liquefaction was found to increase substantially with decreasing values of B.

The value of B has been widely employed to evaluate the state of saturation of laboratory samples owing to its convenience of measurement. However, the use of the B-value test to determine the *in situ* state of saturation is difficult. This difficulty inevitably hinders a development of approaches for evaluating *in situ* liquefaction resistance that takes the saturation effect into account, although this effect has long been recognised. On the other hand, the velocity of compression waves (i.e. P-waves), which has been known to be influenced largely by the degree of saturation and can be measured conveniently in the field, appears to hold potential as an indicator of saturation. Effectiveness of the use of P-wave velocity and Poisson's ratio in identifying *in situ* partially saturated zones has been demonstrated by a borehole array site (Yang & Sato, 2000).

It is therefore the objective of this work to attempt to establish a relationship between the liquefaction resistance of sand and its P-wave velocity so that it may allow more significant interpretations and applications. In doing that, first, an empirical function is developed between the liquefaction resistance and B-value based on cyclic test data. The liquefaction resistance in question is expressed in terms of the number of cycles at a specified cyclic stress ratio to cause liquefaction. Second, a theoretical relationship is introduced between the B-value and the velocity of P-waves. Subsequently, a relationship can be finally established relating the liquefaction resistance to the P-wave velocity.

LIQUEFACTION RESISTANCE IN CONNECTION WITH *B*-VALUE

Two series of cyclic tests, which were conducted respectively by Sherif *et al.* (1977) and Yoshimi *et al.* (1989) are investigated. Both series were carried out using torsional simple shear apparatus on standard sands. The sand tested by Sherif *et al.* (1977) was clean Ottawa sand, and that used by Yoshimi *et al.* (1989) was clean Toyoura sand. The physical properties of the two sands are given in Table 1. Test results under different values of B are shown in Fig. 1, in which the cyclic stress ratio is plotted against the number of cycles causing liquefaction. It is clear that the number of cycles increases considerably as the value of B decreases or correspondingly the degree of saturation decreases.

When the number of cycles at lower values of B is normalised by that at the largest value of B (i.e. full saturation condition) for a specified stress ratio, one may find that the normalised number of cycles plotted on a log scale shows a good linear correlation with B, as shown in Fig. 2. The dashed lines in the same plot, generated using the following empirical function, fit the test results very well:

$$N_{\mathrm{PS}} = N_{\mathrm{FS}} \exp\left(1{\cdot}38 \frac{(1{\cdot}0 - B)}{F}\right) \tag{1}$$

in which N_{PS} is the number of cycles under partially saturated conditions, N_{FS} is the number of cycles under fully saturated conditions, and F is an empirical parameter based on test data, which is found to be dependent upon the cyclic stress ratio, as shown in Fig. 3. Similar trends can also be observed in the cyclic triaxial tests conducted by Chaney (1978).

Table 1. Physical properties of the sands used in cyclic tests

Material	D_{50}: mm	D_{10}: mm	C_{u}	$e_{\max}$	$e_{\min}$	D_{r}	Reference
Ottawa sand	0·40	0·20	2·1	0·76	0·50	Loose state	Sherif *et al.* (1977)
Toyoura sand	0·175	0·129	1·52	0·976	0·605	60%	Yoshimi *et al.* (1989)

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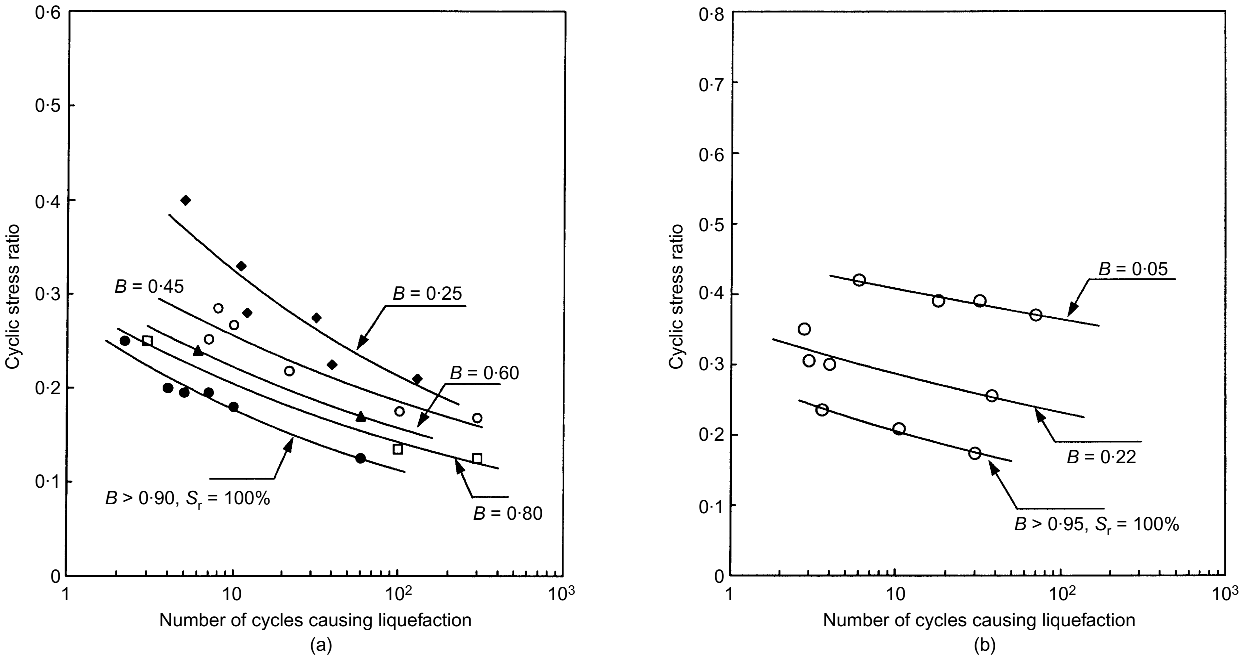

Fig. 1. Laboratory test data for cyclic strength of sands affected by saturation: (a) Ottawa sand (after Sherif *et al.*, 1977); (b) Toyoura sand (after Yoshimi *et al.*, 1989)

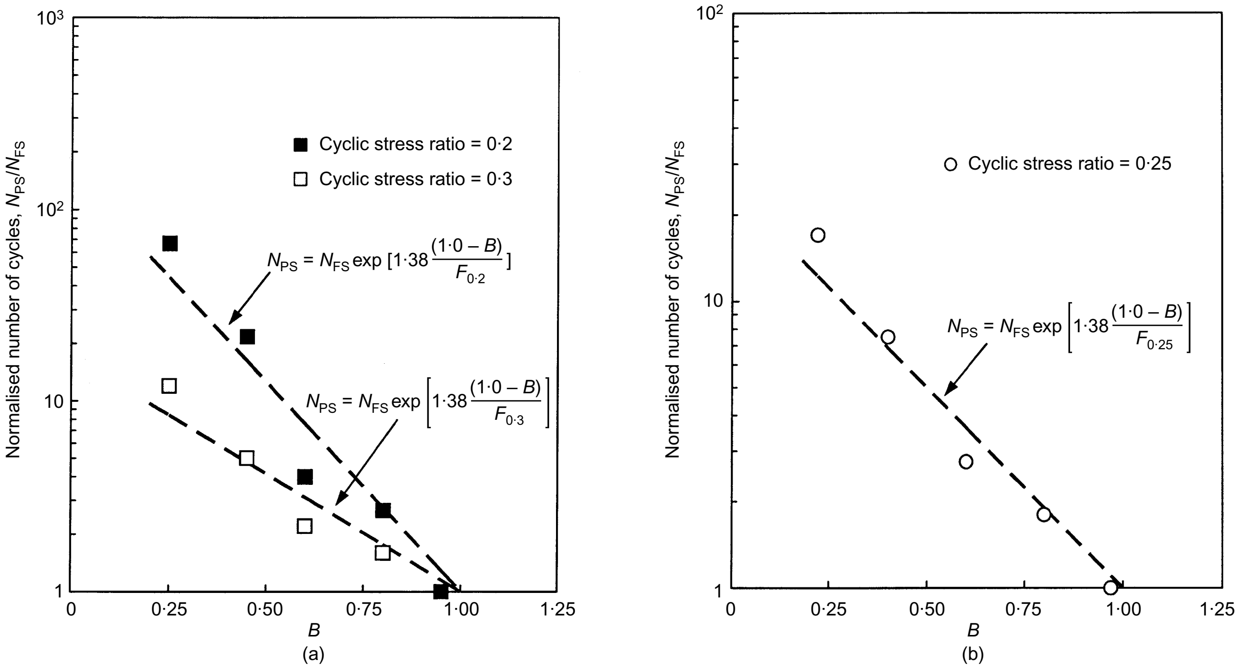

Fig. 2. Normalised number of cycles as a function of *B*: (a) Ottawa sand; (b) Toyoura sand

RELATIONSHIP BETWEEN *B* AND P-WAVE VELOCITY

According to Skempton (1954), *B* is defined as the ratio between the increment of pore pressure and the increment of confining pressure, and can be expressed as

$$B = \frac{1}{1 + n\dfrac{K_b}{K_f}} \tag{2}$$

in which n is porosity, K_b is the bulk modulus of the soil skeleton and K_f is the bulk modulus of the pore fluid. By introducing the dependence of K_f on the degree of saturation, S_r, and the bulk modulus of pore water, K_w, as shown in Yang & Sato (2001), equation (2) can readily be expressed in terms of the degree of saturation as

$$B = \frac{1}{1 + n\dfrac{K_b}{K_w} + n\dfrac{K_b}{p_a}(1 - S_r)} \tag{3}$$

in which p_a is the absolute fluid pressure (i.e. including the atmospheric pressure). It is this expression that forms the basis for *B* being an index of the degree of saturation.

On the other hand, the propagation of P-waves in a

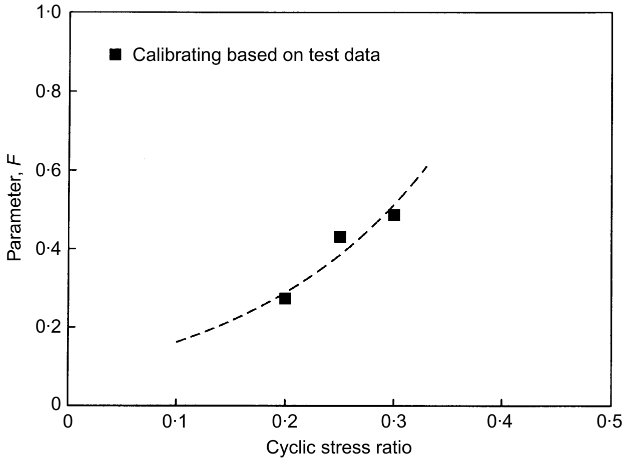

Fig. 3. Dependence of empirical parameter *F* on cyclic stress ratio

partially saturated soil with compressible constituents has been discussed for a wide range of frequencies (Yang & Sato, 1998). At low frequencies of engineering interest, P-wave velocity takes a simplified form as follows, assuming incompressibility of solid grains (e.g. Yang & Sato, 2000):

$$V_\mathrm{P} = \left(\frac{K_\mathrm{b} + 4G/3 + K_\mathrm{f}/n}{\rho}\right)^{1/2} \tag{4}$$

where G is the shear modulus and ρ is the total density, which is expressed in terms of the density of grains, ρ_s, and the density of the fluid, ρ_f, as $\rho = (1 - n)\rho_\mathrm{s} + n\rho_\mathrm{f}$. From equations (2) and (4) an expression relating the P-wave velocity to B can be obtained as follows:

$$V_\mathrm{P} = \left(\frac{4G/3 + K_\mathrm{b}/(1 - B)}{\rho}\right)^{1/2} \tag{5}$$

Based on equations (3) and (4), the values of B and the P-wave velocity for the Toyoura sand used by Yoshimi *et al.* (1989) are computed as a function of degree of saturation and shown in Fig. 4. The shear modulus of the solid skeleton and the relative density of the sand are available from Yoshimi *et al.* (1989). The bulk modulus of the solid skeleton is determined by $K_\mathrm{b} = 2G(1 + \nu)/3(1 - 2\nu)$, where the Poisson's ratio, ν, is assumed to be 0·3. The absolute pressure is taken as being the atmospheric pressure, 98 kPa, since no back-pressure was applied, as indicated by their tests. The bulk modulus of pore water is well known to be 2·2 GPa. Fig. 4 indicates that both B and the P-wave velocity are sensitive to a change of saturation. At full saturation B approaches 1, while the P-wave velocity is over 1500 m/s; but a decrease of complete saturation to 90% may lead to a value of B close to 0·01 with a P-wave velocity as low as 370 m/s. The relationship between P-wave velocity and B for the Toyoura sand is illustrated in Fig. 5 using equation (5).

LIQUEFACTION RESISTANCE IN RELATION TO P-WAVE VELOCITY

Based on the empirical relationship established between the liquefaction resistance and B (equation (1)) and the theoretical relationship between B and the P-wave velocity (equation (5)), a function can be developed relating liquefaction resistance and P-wave velocity as follows, by a straight algebraic manipulation:

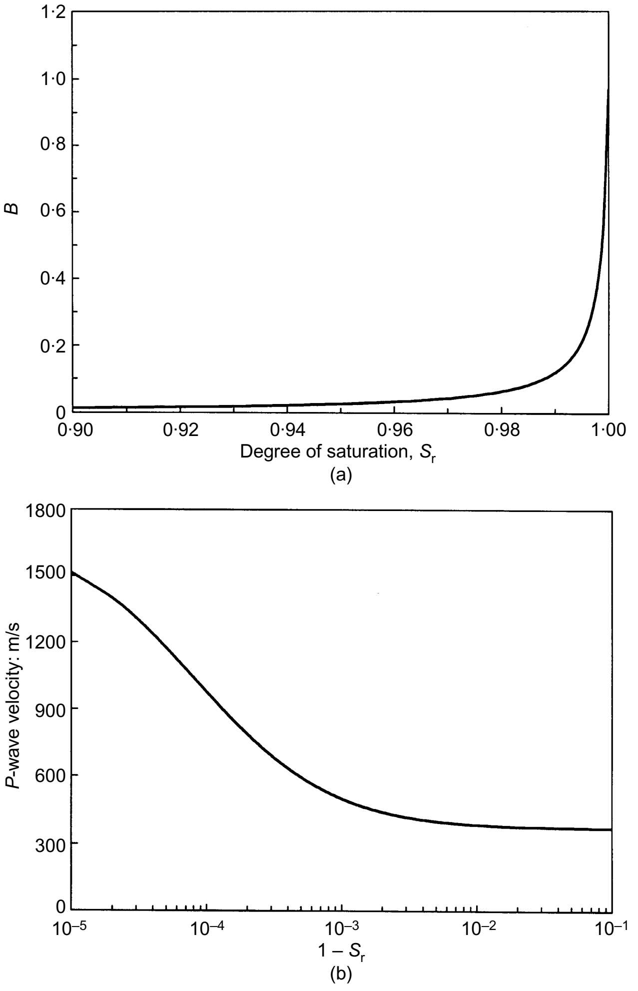

Fig. 4. Variations of (a) *B* and (b) *P*-wave velocity with saturation for Toyoura sand. D_r = 60%, K_b = 165·8 MPa, *G* = 76·5 MPa

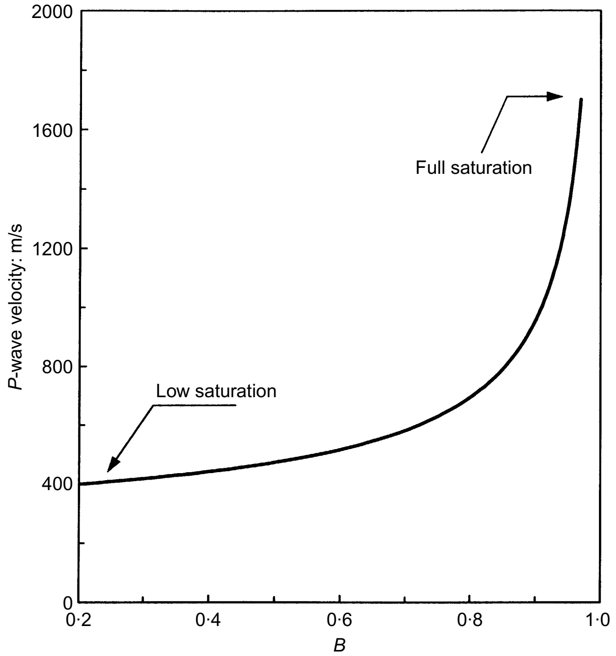

Fig. 5. Relationship between *P*-wave velocity and *B* for Toyoura sand. D_r = 60%, K_b = 165·8 MPa, *G* = 76·5 MPa

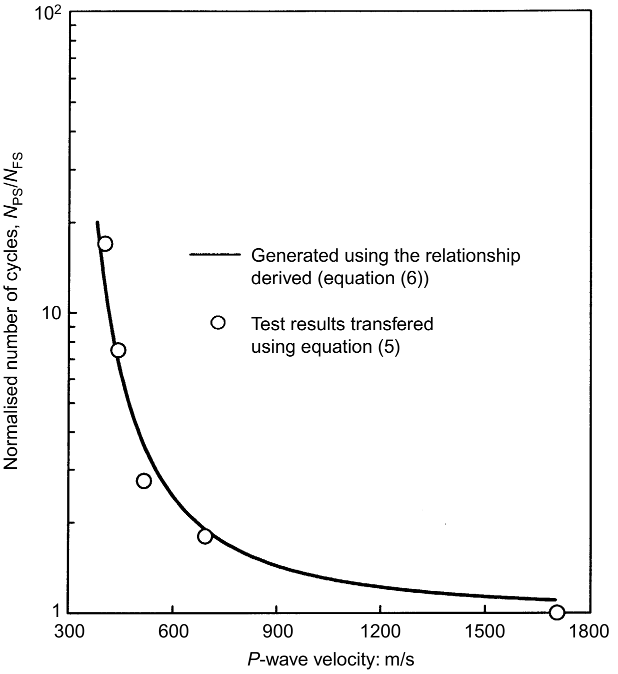

Fig. 6. Normalised number of cycles as a function of *P*-wave velocity for Toyoura sand. $D_r = 60\%$, DA = 5%, stress ratio = 0·25

$$N_{PS} = N_{FS} \exp\left(\frac{1{\cdot}38}{F}\frac{K_b}{\rho V_P^2 - 4G/3}\right) \tag{6}$$

This function can also be given in an alternative form as

$$N_{PS} = N_{FS} \exp\left(\frac{1{\cdot}38}{F}\frac{K_b}{\rho V_P^2 - \rho V_s^2 4/3}\right) \tag{7}$$

where V_s is the velocity of shear waves.

The relationship in equation (6) or equation (7) is of considerable interest because it makes it possible to evaluate the *in situ* liquefaction potential of sands as affected by saturation through the measurements of P-wave velocity. Using this relationship Fig. 6 shows the normalised number of cycles causing liquefaction against the P-wave velocity for the Toyoura sand at a cyclic stress ratio of 0·25. It is clear that the liquefaction resistance correlates with the velocity of P-waves. The number of cycles causing liquefaction at a P-wave velocity of 400 m/s is about 12 times that at a P-wave velocity of 1600 m/s. For the sake of comparison, the test data in terms of B from Yoshimi *et al.* (1989) are transferred here by the relation between B and the P-wave velocity and shown in the same plot. It is apparent that the prediction using the relationship derived agrees reasonably well with the data points. Of course, the data points presented are not true test data since no measurement of P-wave velocity was performed by Yoshimi *et al.* (1989); they are merely illustrative of the way the idea might be used. Further validation of the theoretical development by tests involving the measurement of P-wave velocity is desirable.

CONCLUSIONS

An attempt has been made to develop an explicit relationship between the liquefaction resistance of sand and its P-wave velocity. This relationship has been successfully obtained by introducing a theoretical relationship between the value of B and the P-wave velocity into an empirical function relating the liquefaction resistance to B, established based on laboratory test data. The significance of the relationship is that it makes it possible to evaluate the *in situ* liquefaction resistance of sands as affected by saturation through the measurements of P-wave velocity. Further improvements and developments based on more test results are of interest and can be readily made.

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NOTATION

B	pore pressure coefficient
F	empirical parameter
G	shear modulus
K_b	bulk modulus of soil skeleton
K_f	bulk modulus of pore fluid
K_w	bulk modulus of pore water
n	porosity
N_{FS}	number of cycles causing liquefaction under fully saturated conditions
N_{PS}	number of cycles causing liquefaction under partially saturated conditions
p_a	absolute pore pressure
S_r	degree of saturation
V_p	velocity of compression waves
V_s	velocity of shear waves
ν	Poisson's ratio
ρ	total density
ρ_f	density of pore fluid
ρ_s	density of solid grains

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