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TECHNICAL NOTE

Non-uniqueness of flow liquefaction line for loose sand

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KEYWORDS: failure; laboratory tests; liquefaction; sands

INTRODUCTION

It has been consistently observed in undrained triaxial tests (e.g. Casagrande, 1971; Castro *et al.*, 1982; Vaid & Chern, 1985; Ishihara, 1993) that, subjected to monotonic loading, very loose sand exhibits a peak strength at a small shear strain and then collapses to flow rapidly to large strains at low effective confining pressure and low strength, as illustrated in Fig. 1. Furthermore, there exists an ultimate state of shear failure at which the sand flows continuously under constant stress and constant volume. The ultimate state, termed as steady state (Poulos, 1981), is essentially the same as the well-known critical state (Roscoe *et al.*, 1958). The type of behaviour described above is now recognised as flow liquefaction, which may produce the most devastating effects of all the liquefaction-related phenomena. Flow liquefaction failures are characterised by a sudden loss of strength and a rapid development of large deformation. The collapse of Lower San Fernando Dam (Seed *et al.*, 1975) is a typical example of such failures.

Owing to its dramatic effects and its complex nature, considerable efforts have been made to understand and characterise flow liquefaction. Sladen *et al.* (1985) proposed the concept of a collapse surface based on some triaxial test results. In this concept, for a series of specimens initially consolidated at the same void ratio at different confining pressures, the locus of peak points in the effective stress paths is a straight line that projects linearly through the steady-state point in $q - p'$ space (see Fig. 2(a)), where $p' = (\sigma_1' + 2\sigma_3')/3$ is mean effective stress and $q = \sigma_1 - \sigma_3$ is deviator stress in a triaxial setting. In $q - p' - e$ space, where e is void ratio, these points form a space that passes through the steady- or critical-state line. The collapse surface or line concept is fundamentally an extension of the steady-state concept and in many respects follows the principles of critical-state soil mechanics (Schofield & Wroth, 1968). The key point of this method is that the collapse line is unique: the parameters describing its position (the slope and the intercept on the vertical axis) in the stress path space can be converted into parameters analogous to Mohr–Coulomb failure parameters, and may therefore be used in conventional limit equilibrium stability analysis.

The collapse line concept has been recognised by some researchers. For example, Ishihara (1993) showed the existence of such a unique collapse line in the stress path space for loose Toyoura sand. On the other hand, Vaid & Chern (1985), Lade (1993) and some recent experimental investigations have assumed that the locus of peak points in the effective stress paths is a straight line, the flow liquefaction line, that passes through the origin rather than through the steady state, as schematically shown in Fig. 2(b). The flow liquefaction surface or line, which also assumes the uniqueness of the line in stress path space, seems to receive more recognition (Kramer, 1996).

In this study a new interpretation, a *flow liquefaction line* varying with the state of soil rather than unique in the stress path space, is proposed, based on careful examination of experimental data. The flow liquefaction line is defined here as a line that connects the peak point in any one single stress path with the stress origin. Within the framework of critical-state soil mechanics, a dependence of the slope of the flow liquefaction line upon a state parameter that simultaneously accounts for the stress level and soil density is suggested.

EXPERIMENTAL EVIDENCE

The triaxial test data analysed are from Castro *et al.* (1982), Sladen *et al.* (1985) and Ishihara (1993). In Table 1 the index properties of the sands tested (Banding sand No. 6,

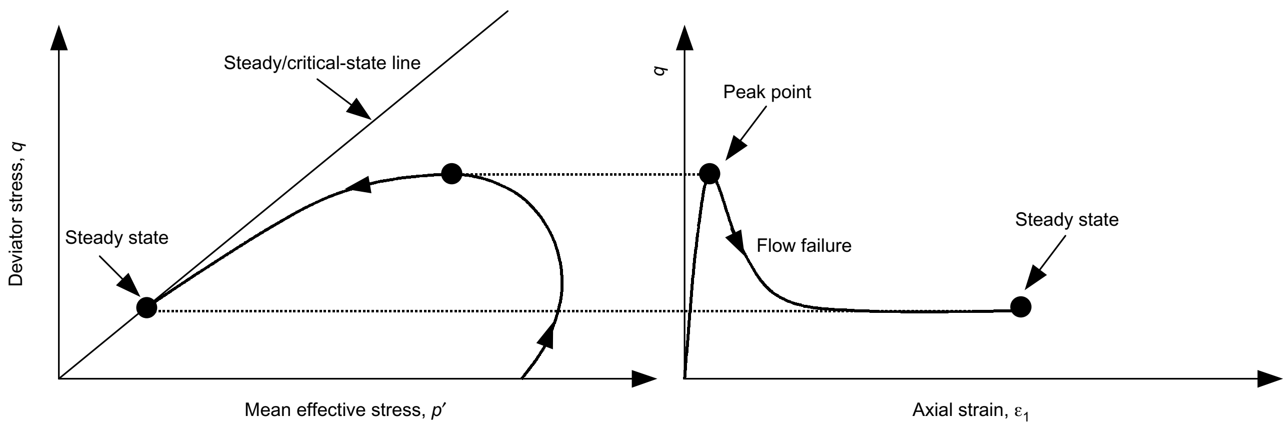

Fig. 1. Flow liquefaction of loose sand in undrained monotonic loading tests

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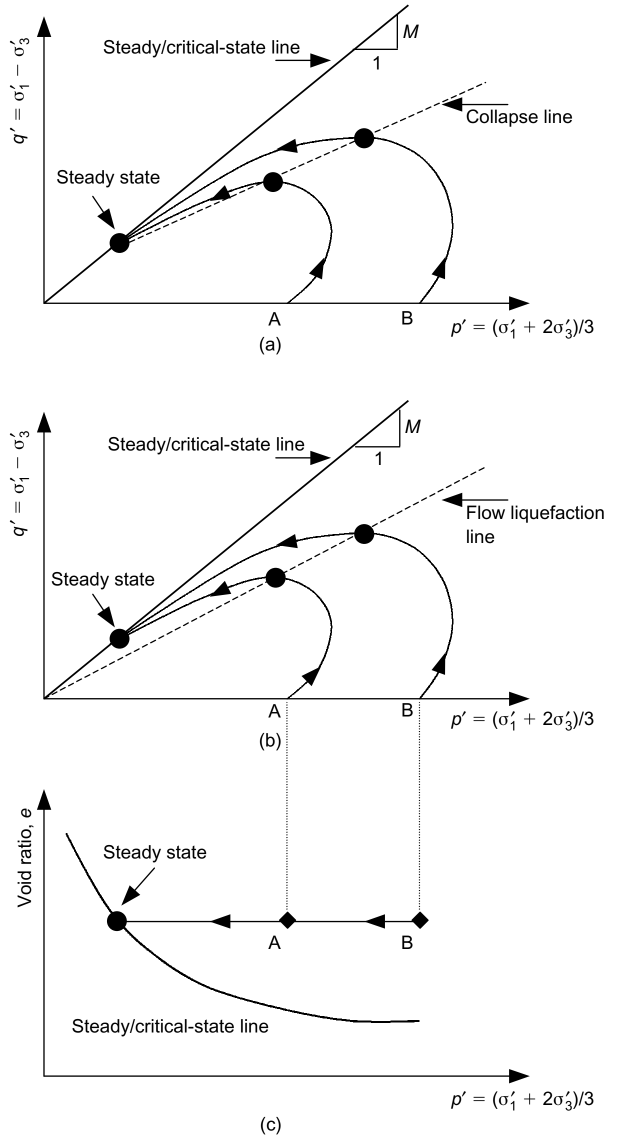

Fig. 2. Schematic diagram of: (a) collapse line concept; (b) flow liquefaction line concept

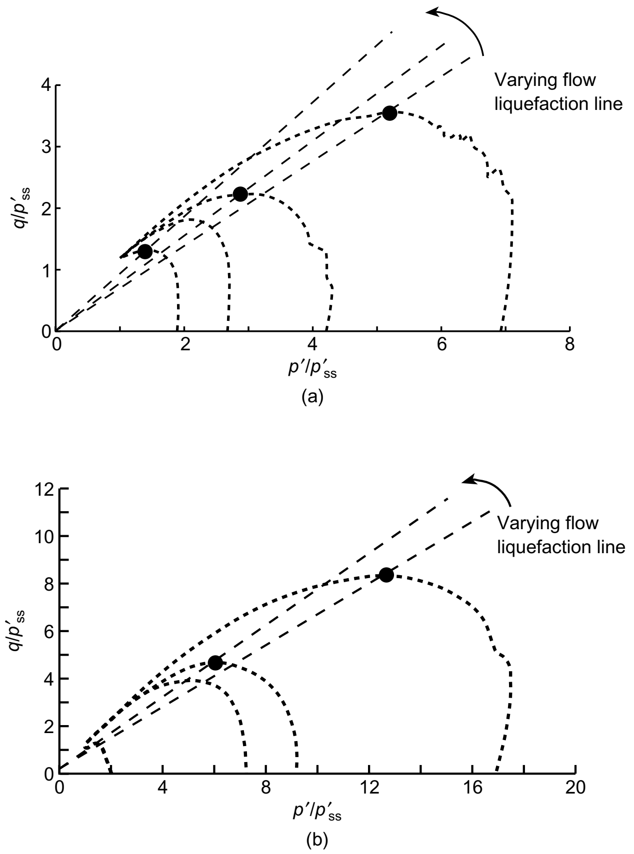

Fig. 3. Test results for (a) Leighton Buzzard sand and (b) Nerlerk sand in stress path space (data from Sladen *et al.*, 1985)

Leighton Buzzard sand, Nerlerk sand and Toyoura sand) are summarised. Fig. 3 shows the test results for the Leighton Buzzard sand and Nerlerk sand in the stress path space. Both the deviator stress and the mean effective stress were normalised by the mean stress at the steady/critical state at the same void ratio, p'_{ss} (Sladen *et al.*, 1985). It is evident that the peak points in the stress paths do not lie on a single line through the origin, but rather, the flow liquefaction line varies with the stress level. It is found that the slope of the flow liquefaction line increases with decreasing confining pressure, and the critical-state line provides an upper bound.

The evidence of a moving flow liquefaction line and the similar tendency of the variation of the slope of the line with confining pressure are also clearly observed in Figs 4 and 5 for Banding sand and Toyoura sand respectively.

STATE-DEPENDENT FLOW LIQUEFACTION LINE

A proper evaluation of the behaviour of the varying flow liquefaction line with different initial states of soil is of value. The nature of the steady- or critical-state line implies the limited applicability of absolute measures of density, such as void ratio, for characterising a potentially liquefiable soil. The behaviour of a cohesionless soil should be more closely related to the proximity of its initial state to the critical-state line. With the critical state as a basis, a state parameter (Been & Jefferies, 1985) can be defined as

$$\psi = e - e_c \qquad (1)$$

where e is the void ratio at the initial state and e_c is the void ratio at the critical state under the same mean effective stress, as illustrated in Fig. 6. The state parameter is a measure of how far the material state is from the critical

Table 1. Index properties of sands tested

Sand	D_{50}: mm	C_u	e_{max}	e_{min}	F_c: %	e	Reference
Banding No. 6	0·157	1·70	0·82	0·52	–	Loose state	Castro *et al.* (1982)
Leighton Buzzard	0·86	1·16	0·75	0·58	0	Loose state	Sladen *et al.* (1985)
Nerlerk	0·28	2·0	0·94	0·62	2	Loose state	Sladen *et al.* (1985)
Toyoura	0·17	1·7	0·977	0·597	0	0·908	Ishihara (1993)

Note: D_{50} = mean grain size; C_u = uniformity coefficient; F_c: % = fines content; e = void ratio

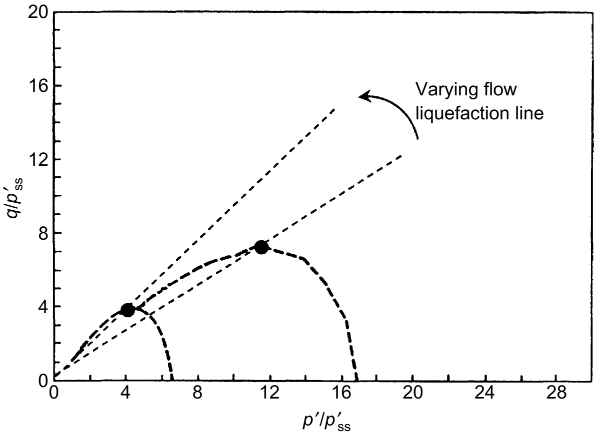

Fig. 4. Test results for Banding sand in stress path space (data from Castro *et al.*, 1982; also see Sladen *et al.*, 1985)

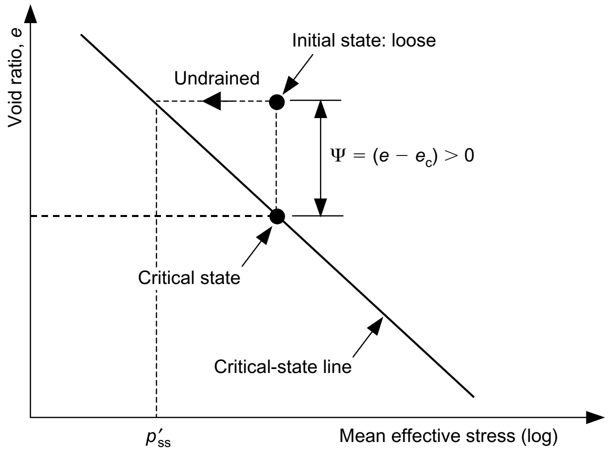

Fig. 6. Definition of state parameter

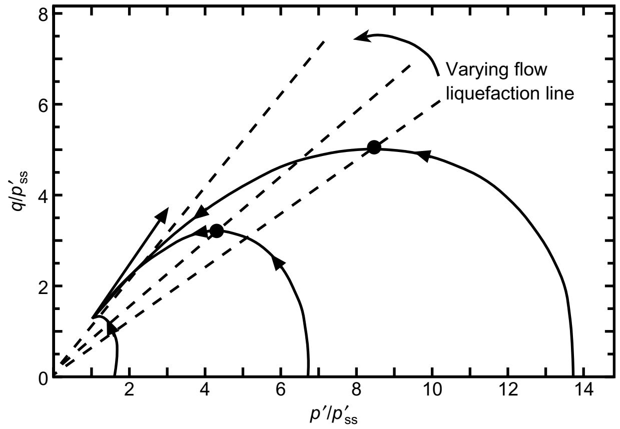

Fig. 5. Test results for Toyoura sand in stress path space (data from Ishihara, 1993)

state in terms of density. When ψ is positive the soil is in a loose state that is susceptible to liquefaction.

For laboratory tests, the state parameter ψ at the initial state can be conveniently estimated provided the critical-state line is determined in the $e - p'$ plane. It is worth mentioning here that it is simply a matter of mathematical convenience that the critical-state line is usually assumed as being linear in a semi-log form. Alternative representations of the line on a different scale may be made to better fit experimental data: this is the case when a wide range of stresses is considered (e.g. Verdugo & Ishihara, 1996).

Figure 7 shows the stress ratio, q/p', at the peak points in the stress paths as a function of the state parameter for the Leighton Buzzard sand (Fig. 3(a)) and Toyoura sand (Fig. 5). It is clear that a correlation exists between the two variables. For the data analysed, an exponential function as suggested below can describe the relationship between q/p' and ψ reasonably well for both sands:

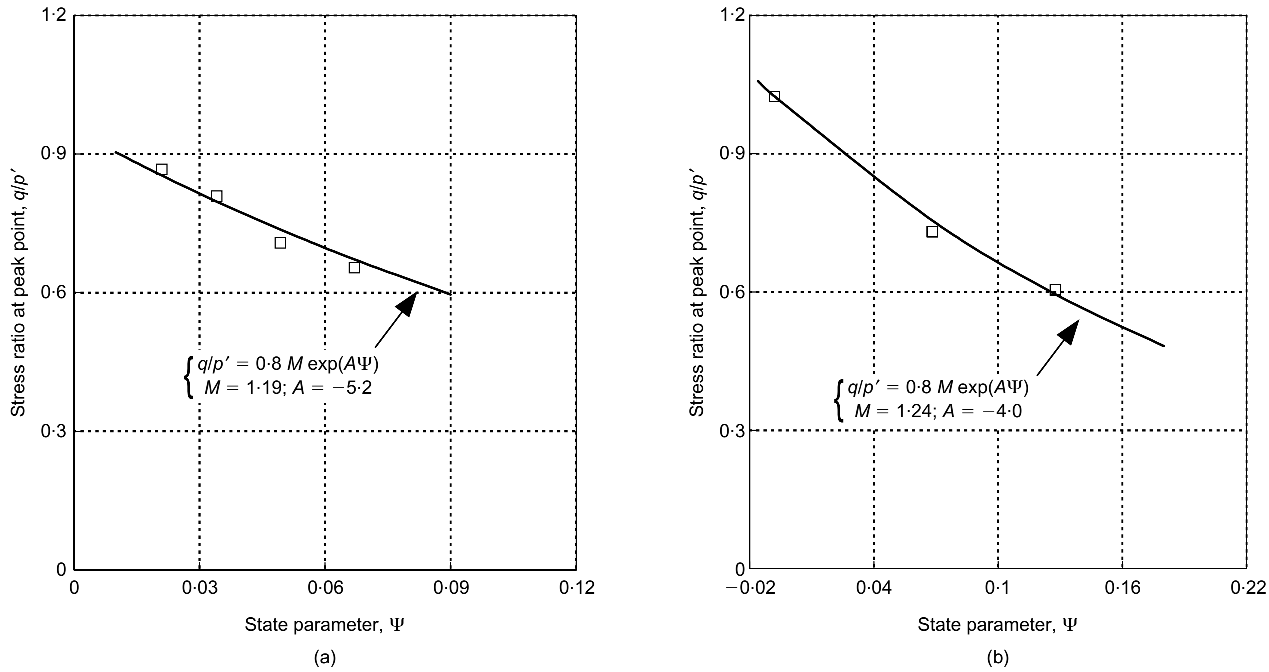

Fig. 7. Relationship between peak stress ratio and state parameter: (a) Leighton Buzzard sand (data from Sladen *et al.*, 1985); (b) Toyoura sand (data from Ishihara, 1993)

$$\left(\frac{q}{p'}\right)_{\text{peak}} = 0 \cdot 8M \exp(A\psi) \tag{2}$$

in which M is the stress ratio q/p' at critical state and A is a parameter that is less than zero. For Leighton Buzzard sand the critical stress ratio M is 1·19 (Sladen *et al.*, 1985), and A is calibrated as −5·2. For Toyoura sand M is 1·24 (Ishihara, 1993), and an appropriate A is found to be −4·0.

Note that the relationship in equation (2) is established based on limited experimental data, and an alternative fit might be possible. Provided more data of quality are available, the relationship could be improved. For this reason, a more general relationship may take the following form:

$$\left(\frac{q}{p'}\right)_{\text{peak}} = \frac{M}{B} \exp(A\psi) \tag{3}$$

where B is an additional parameter that is expected to vary within a narrow range. As the critical-state line serves as the upper bound for the flow liquefaction line (see Figs 3–5), the value of B is required to satisfy the condition $B \geqslant 1{\cdot}0$.

The state parameter ψ has been related to CPT resistance and other *in situ* test results (Been, 1998), and therefore the relationship established in equation (3) is of interest in regard to its potential applications in engineering practice. Based on equation (3), furthermore, a friction angle ϕ_{FLL} for characterising the slope of the flow liquefaction line (FLL) can be introduced as

$$\sin\phi_{\text{FLL}} = \frac{3\left[\dfrac{M}{B}\exp(A\psi)\right]}{6 + \left[\dfrac{M}{B}\exp(A\psi)\right]} \tag{4}$$

It can be readily shown that the flow liquefaction angle decreases with increasing state parameter: that is, the looser the sand the smaller the flow liquefaction angle. The flow liquefaction line separates the liquefaction process into stable and unstable states in the stress path space. As a result, if the stress conditions in an element of soil reach this line, flow liquefaction is to be triggered and the shear resistance will be reduced rapidly to the critical-state strength.

CLOSING REMARKS

Owing to the complex nature and devastating effects of the flow liquefaction of loose sand, its proper characterisation has been, and continues to be, a very challenging topic. The collapse line (Sladen *et al.*, 1985; Ishihara, 1993) and flow liquefaction line (Vaid & Chern, 1985; Lade, 1993) are two approaches that have been in widespread use. Both approaches assume that the locus of peak points in the effective stress paths is a unique line that passes through either the steady-state point or the origin in stress path space. The slope of the collapse or flow liquefaction line is hence treated as a material constant irrespective of the state of soil.

In this study a new concept has been proposed that states that the flow liquefaction line is not unique but rather varies with the state of soil. Experimental evidence for different sands was shown to clearly support this interpretation. Within the framework of critical-state soil mechanics, an explicit relationship was suggested between the slope of the flow liquefaction line and the state parameter that accounts for both stress level and the density of soil. The present work clarifies the confusion and contradiction related to the collapse line concept and the flow liquefaction line concept. In particular, it provides a useful framework for conceptual understanding of the complicated behaviour of liquefiable soil both before and after liquefaction. Obviously, more experimental data of high quality are needed to further validate this new interpretation and to improve the relationship suggested.

ACKNOWLEDGEMENTS

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NOTATION

- A, B parameters used in the relationship in equation (3)
- e void ratio
- e_c critical void ratio
- M critical stress ratio
- p' mean effective stress
- p'_{ss} mean effective stress at critical state
- q deviator stress
- ϕ_{FLL} friction angle for characterising the flow liquefaction line
- ψ state parameter

REFERENCES

Been, K. (1998). The critical state line and its application to soil liquefaction. In *Physics and mechanics of soil liquefaction*, pp. 195–204. Rotterdam: Balkema.

Been, K. & Jefferies, M. G. (1985). A state parameter for sands. *Géotechnique* **35**, No. 2, 99–112.

Casagrande, A. (1971). On liquefaction phenomena. *Géotechnique* **21**, No. 3, 197–202.

Castro, G., Enos, J. L., France, J. W. & Poulos, S. J. (1982). *Liquefaction induced by cyclic loading*, Report No. NSF/CEE-82018. Washington, DC: National Science Foundation.

Ishihara, K. (1993). Liquefaction and flow failure during earthquakes. *Géotechnique* **43**, No. 3, 351–415.

Kramer, S. L. (1996). *Geotechnical earthquake engineering*. Englewood Cliffs, NJ: Prentice Hall.

Lade, P. V. (1993). Initiation of static instability in the submarine Nerlerk berm. *Can. Geotech. J.* **30**, No. 6, 895–904.

Poulos, S. J. (1981). The steady state of deformation. *J. Geotech. Engng Div., ASCE* **107**, No. 5, 553–562.

Roscoe, K. H., Schofield, A. N. & Wroth, C. P. (1958). On the yielding of soils. *Géotechnique* **8**, No. 1, 22–52.

Schofield, A. N. & Wroth, C. P. (1968). *Critical state soil mechanics*. London: McGraw-Hill.

Seed, H. B., Lee, K. L., Idriss, I. M. & Makdisi, F. I. (1975). The slides in the San Fernando Dams during the earthquake of February 9, 1971. *J. Geotech. Engng Div., ASCE* **101**, No. 7, 651–688.

Sladen, J. A., D'Hollander, R. D. & Krahn, J. (1985). The liquefaction of sands, a collapse surface approach. *Can. Geotech. J.* **22**, No. 4, 564–578.

Vaid, Y. P. & Chern, J. C. (1985). Cyclic and monotonic undrained response of saturated sands. In *Advances in the Art of Testing Soils under Cyclic Conditions*, pp. 120–147. American Society of Civil Engineers.

Verdugo, R. & Ishihara, K. (1996). The steady state of sandy soils. *Soils Found.* **36**, No. 2, 81–91.

Yang, J. (2004). *Géotechnique* **54**, No. 1, 66–68

DISCUSSION

Non-uniqueness of flow liquefaction line for loose sand

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M. Jefferies and K. Been, *Golder Associates, Vancouver and Houston*

A relationship is proposed for the stress ratio η_L at which flow liquefaction instability may arise with loose sand in terms of the sand's state parameter. The usage of the state parameter is in terms of the initial sample void ratio and initial stress state. Formally writing ψ_0, where the subscript '0' clarifies definition using initial values, the author proposes that:

$$\eta_L = \frac{M}{B}\exp(A\psi_0) \tag{3}$$

where B is a soil property with a common value of 1·25 (= 1/0·8) in the case of Leighton Buzzard and Toyoura sand.

Theoretical interest centres on the appearance of B in equation (3). For a not so loose sand with $\psi_0 = 0$, equation (3) implies that soil deforms plastically and indefinitely at a significantly lower shear stress than that corresponding to critical conditions for any $B > 1$. To the discusser's knowledge, equation (3) is the first proposal of a non-unique stress state for critical yielding. Although the idea of non-unique critical void ratios has arisen in the literature, M in triaxial compression has always been treated as unique for any sand.

An alternative to this new view of critical stress conditions is that B has arisen because of the difference between ψ and ψ_0. The state parameter ψ is a general measure, and its application in constitutive models is as a variable, not an initial index parameter (e.g. Jefferies, 1993; Wood *et al.*, 1994; Manzari & Dafalias, 1997; Li & Dafalias, 2000). For undrained tests, the difference between ψ and ψ_0 arises only through changes in stress. Assuming the usual semi-log representation of the CSL with slope λ, the two measures are related by

$$\psi = \psi_0 + \lambda\log(p'/p'_0) \tag{4}$$

Figure 8 shows equation (4) applied to the Leighton Buzzard and Toyoura sand data in the note using previously

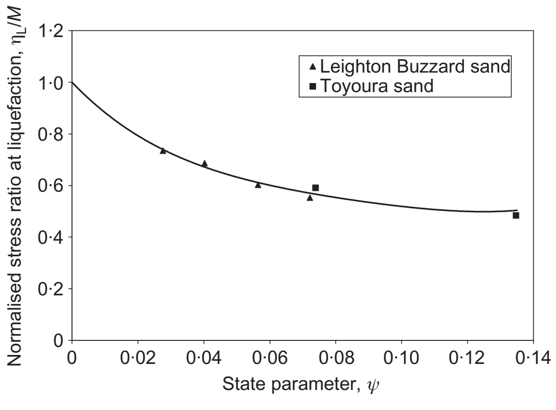

Fig. 8. Stress ratio at peak undrained strength of loose sand expressed in terms of *current* state parameter ψ at onset of liquefaction

published values of λ, plotting the ratio η_L/M to normalise the data. A trend line has been drawn and extrapolated back to $\eta_L/M = 1$ for the condition $\psi = 0$. This trend line is a plausible fit to the data and preserves a unique critical state. Although a single trend fits Leighton Buzzard and Toyoura sands, these are predominantly quartz soils and rather similar. Fig. 9 shows data for a wider range of sands, and supports the form of the trend drawn through the author's data in Fig. 8, although differences from sand to sand now become apparent.

Also shown in Fig. 9 is a trend line from extrapolating limiting stress ratios in drained triaxial tests on dense sand. Been & Jefferies (1985) reported a near-unique relationship between ϕ_{max} and ψ_0. These data can be transformed to a limiting stress ratio, η_{max}, at ψ. Extrapolating the average trend for η_{max} at ψ from the $\psi \ll 0$ regime of the dense tests, by recognising that η_{max} and η_L are similar in concept, gives the line shown in Fig. 9. This dense drained limiting stress ratio sensibly bounds the stress ratios at the onset of liquefaction, although the differences between the loose sand data and the extrapolated limiting ratio from dense tests may be practically significant. An updated form of equation (3) would be useful, respecting the condition that

$$\psi = 0 \Rightarrow \eta_L/ = 1$$

A difficulty in representing peak s_u data at onset of liquefaction as a ratio, η_L, is that the pore pressure is changing rapidly at peak s_u, and an accurate determination of p' is problematic. There are issues of transducer time lag with load-controlled tests, and strain-controlled tests have increased excess pore water pressures from sample creep (Leong *et al.*, 2000). These factors suggest caution in relying on η_L, and perhaps an undrained strength ratio (s_u/p'_0) approach following Bishop (1971) is more appropriate in practice.

Author's reply

The author thanks the discussers for their interest in the topic and their valuable comments. They offer an alternative interpretation that uses the state parameter at peak strength rather than the initial state parameter ψ_0 used in the author's proposal. In general the trend shown in Fig. 8 is quite similar to that presented in terms of ψ_0, and it is of particular interest to notice that the data shown in Fig. 9 for a wider range of sands support the form of the trend. In what follows, the author would like to clarify several points.

(*a*) The critical state line (CSL) is usually represented by a linear relationship in a semi-log form. This representation makes it convenient to relate the state parameter ψ as a variable with its initial value. Some experimental results have, however, indicated that the CSL for sands is not a straight but a curved line on the semi-log scale (e.g. Verdugo & Ishihara, 1996). For this reason, caution should be exerted when using the linear relationship equation (4) to locate the values of ψ at specific states. Alternative representations of the CSL on a different scale may need to be made to fit the experimental data better.

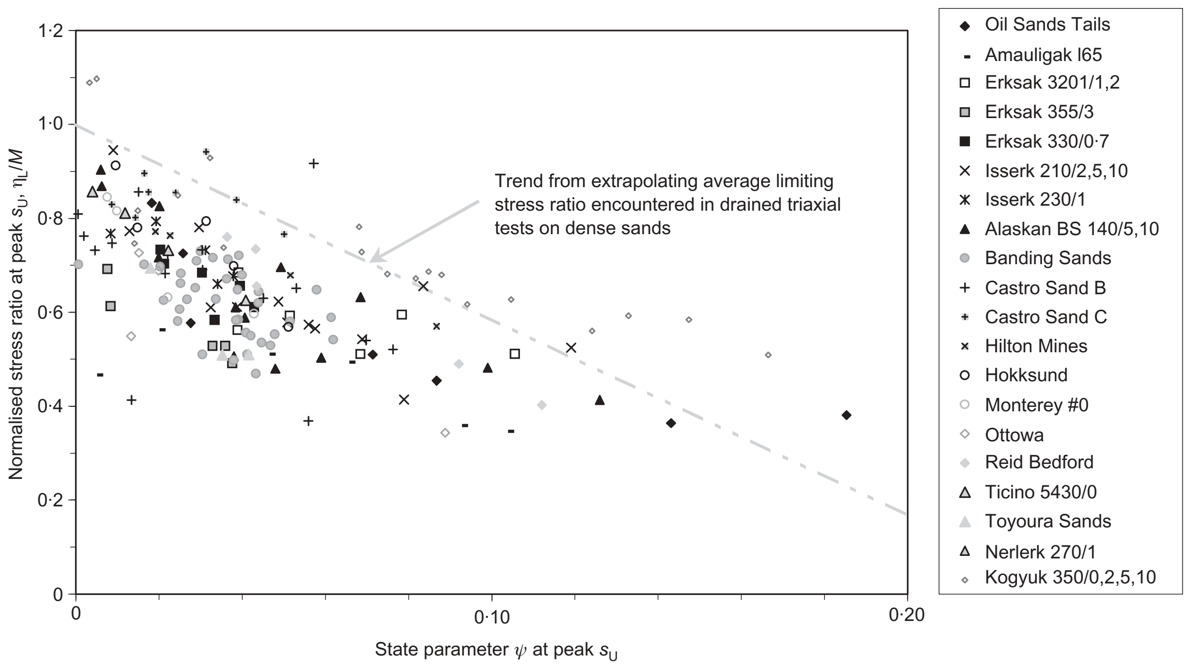

Fig. 9. General trend in stress ratio at peak strength undrained strength for sands

(*b*) The state parameter is a measure of how far the material state is from the critical state in terms of density. Its advantage has been recognised by its application in constitutive modelling of sand behaviour. From the point of view of constitutive modelling, the state parameter ψ is an internal variable that can be incorporated in the formulation, as correctly pointed out by the discussers. To illustrate the role of state parameter in such applications, Fig. 10 shows the evolution of the state parameter during the deformation of Toyoura sand subject to undrained triaxial compression, and Fig. 11 presents the relationship between the dilatancy, a key issue in studying the behaviour of sands, and the state parameter as a variable. The responses are obtained within the framework of state-dependent modelling (Li & Dafalias, 2000), and the dilatancy is defined as

$$d = \frac{\mathrm{d}\,\varepsilon_v^p}{\mathrm{d}\,\varepsilon_q^p}$$

the ratio of plastic volumetric strain increment to

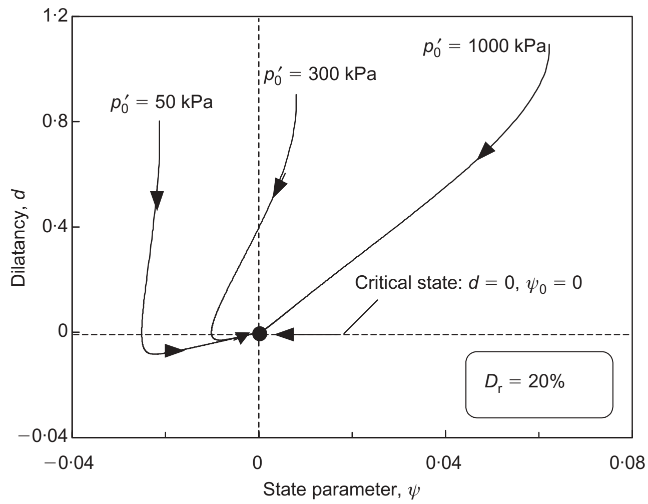

Fig. 11. Relationship between dilatancy and state parameter during the deformation of Toyoura sand subject to undrained triaxial compression

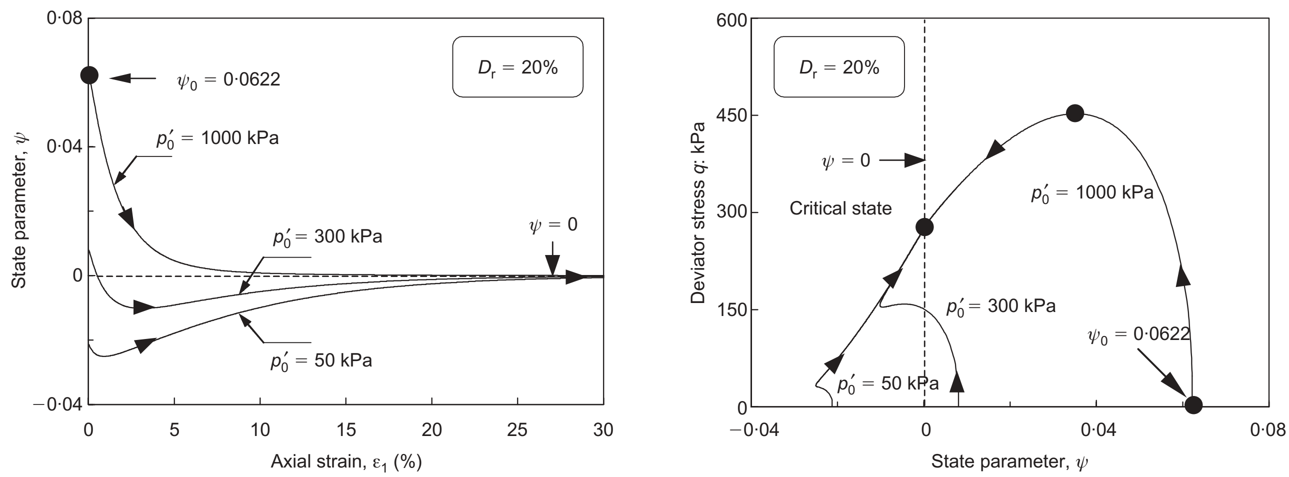

Fig. 10. Evolution of state parameter during deformation of Toyoura sand subject to undrained triaxial compression

plastic deviator strain increment. It is worthwhile noting that the manner of the evolution of the state parameter with the deviator stress is very similar to that followed by the stress paths in terms of q and p'.

(*c*) If it is the case that the state parameter ψ is incorporated in the constitutive formulation as an internal variable, mathematically it is even possible to derive an analytical relationship between the peak strength and the corresponding state parameter, as demonstrated in Yang & Li (2004), where a unique relationship has been established between the drained peak friction angle, ϕ'_p, and the state parameter at peak, ψ_p (see Fig. 12). In engineering practice it would, however, be difficult to determine ψ_p accurately. As such, the peak friction angle has also been presented in terms of the initial state parameter, ψ_0, for the purpose of practical applications (Yang & Li, 2004). The initial state parameter describes the material state before the deformation occurs. It is apparent from Fig. 12 that both relationships (indicated by the solid and dashed lines respectively) exhibit a similar trend, although differences exist between them.

(*d*) Some problems may appear with an accurate determination of the mean effective stress at onset of liquefaction in the laboratory, as remarked by the discussers. It is also noted, on the other hand, that quality test data can be obtained with the development of testing techniques (Verdugo & Ishihara, 1996; Vaid *et al.*, 2001). The proposed approach has its advantage in that it establishes a relationship for the stress ratio at onset of liquefaction in terms of the initial state parameter, which simultaneously accounts for the initial void ratio and initial stress level, and has been related to some in-situ test results (e.g. CPT resistance). The valuable data provided by the discussers for a wider range of sands also indicate the potential of the approach in practice.

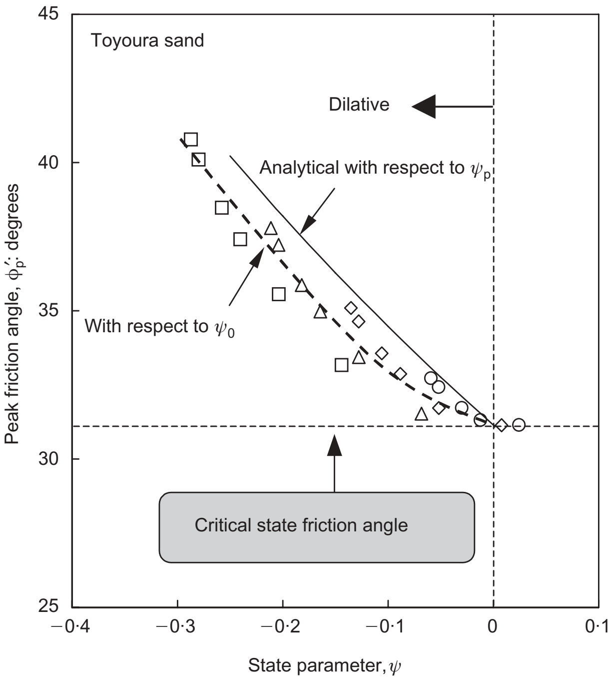

Fig. 12. Relationship between drained peak friction angle and state parameter

REFERENCES

Bishop, A. W. (1971). Shear strength parameters for undisturbed and remoulded soil specimens. *Stress–strain behaviour of soils: Proceedings of the Roscoe Memorial Symposium* (ed. R. H. G. Parry), pp. 3–58. Henley-on-Thames: G.T. Foulis.

Jefferies, M. G. (1993). NorSand: a simple critical state model for sand. *Géotechnique* **43**, No. 1, 91–103.

Leong, W. K., Chu, J. & Teh, C. I. (2000). Liquefaction and instability of a granular fill material. *Geotech. Test. J.* **23**, No. 2, 178–192.

Li, X. S. & Dafalias, Y. F. (2000). Dilatancy for cohesionless soils. *Géotechnique* **50**, No. 4, 449–460.

Manzari, M. T. & Dafalias, Y. F. (1997). A critical state two-surface plasticity model for sands. *Géotechnique* **47**, No. 2, 255–272.

Vaid, Y. P., Eliadorani, A., Sivathayalan, S. & Uthayakumar, M. (2001). Laboratory characterization of stress–strain behavior of soils by stress and/or strain path loading. *Geotech. Test. J.* **24**, No. 2, 200–208.

Verdugo, R. & Ishihara, K. (1996). The steady state of sandy soils. *Soils Found.* **36**, No. 2, 81–91.

Wood, D. M., Belkheir, K. & Liu, D. F. (1994). Strain softening and state parameter for sand modelling. *Géotechnique* **44**, No. 2, 335–339.

Yang, J. & Li, X. S. (2004). State-dependent strength of sands from the perspective of unified modeling. *J. Geotech. Geoenviron. Engng, ASCE* **130**, No. 2 (in press).