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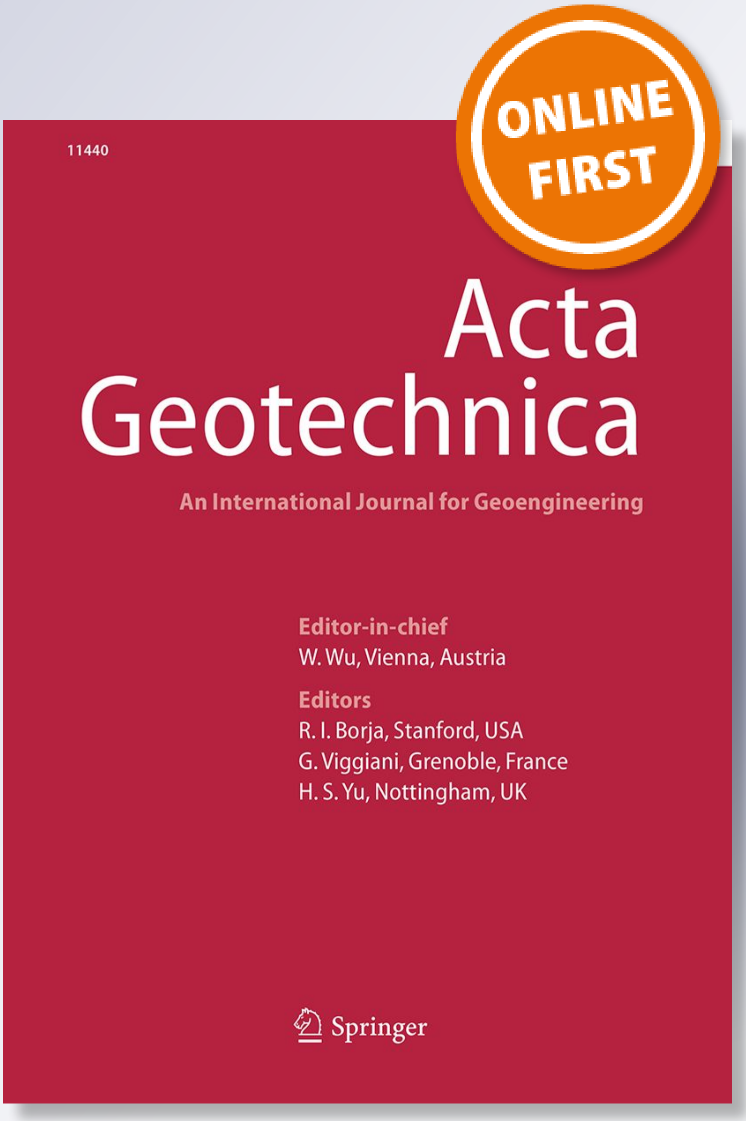

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RESEARCH PAPER

The critical state friction angle of granular materials: does it depend on grading?

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Abstract Whether the critical state friction angle of granular materials depends on grading is a fundamental question of both academic and practical interest. The present study attempts to address this question through a specifically designed experimental program where the influence of particle grading was carefully isolated from other influencing factors. The laboratory experiments show that under otherwise similar conditions, the angle of friction at critical state is a constant independent of grading, but, for a given grading, the angle of friction at critical state is highly dependent on particle shape. This finding suggests that the commonly adopted practice of separately allowing for the effect of particle shape and the effect of grading on critical state friction angle is conceptually inappropriate and, hence, should be taken with caution in geotechnical design to avoid the risk of underestimating safety requirements. The study also reveals that varying particle gradation can impose a marked impact on liquefaction susceptibility of granular soils: Under the same post-consolidation state in terms of void ratio and confining pressure, a well-graded soil tends to be more susceptible to liquefaction than a uniformly graded soil. This variation of liquefaction susceptibility is shown to be consistent with the variation of location of the critical state locus in the compression space and is explainable by the critical state theory.

Keywords Angle of friction · Critical state · Granular soils · Liquefaction susceptibility · Particle grading · Particle shape

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1 Introduction

The angle of shearing resistance at critical state, or conventionally known as critical state friction angle (ϕ_{cs}), is a fundamental soil property that plays an important role in a great variety of geotechnical applications, ranging from foundations and retaining structures to embankments and slopes. It is also a key parameter in formulating constitutive models in the framework of critical state soil mechanics, in which it is usually related to a critical stress ratio (M) in triaxial stress space as follows [9, 26, 37]:

$$M = \frac{6 \sin \phi_{cs}}{3 - \sin \phi_{cs}}. \quad (1)$$

Varying the stress ratio M may affect the yield surface and flow rule and hence the overall stress–strain behavior, as shown in many Cam-clay-type models [45, 46]. Considerable efforts have thus been made to measure and characterize this strength property and the associated stress–dilatancy relation [2, 8, 22, 23, 34, 42]. For granular soils, there is now a general agreement that the critical state friction angle depends on particle shape and grading in addition to mineralogy, as documented in codes of practice and textbooks (e.g., [4, 14, 17]). In particular, the importance of particle grading on soil behavior has received considerable attention in soil mechanics, and it has become a routine practice to determine for given soils their size distribution curves.

Recent notable work on critical state friction angle and stress–dilatancy behavior includes that by Simoni and Houlsby [27], who conducted a series of large-scale direct shear tests on sand–gravel mixtures. They observed that an increase in gravel fraction causes an increase in the critical state friction angle, leading them to conclude that the critical state friction angle is related to particle size

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distribution. By plotting their test data against the coefficient of uniformity (C_u), as shown in Fig. 1a, a trend can be observed that the value of ϕ_{cs} tends to increase with C_u, but the correlation does not appear to be strong. A similar observation was made by Kokusho et al. [12] from undrained triaxial tests on three river sands of different gradings, as shown in Fig. 1b. Compared with typical values reported in the literature for quartz sands, the measured ϕ_{cs} values for these river sands appear to be markedly higher. Note that in both studies the tested soils were hard-grained materials and no significant particle crushing was observed.

Some other studies in the literature, however, seem to suggest a different influence of grading. For example, for a variety of lagoon soils ranging from silty clay and silt to fine and medium sand, Simonini et al. [28] observed from undrained triaxial tests that the critical state friction angle depends not only on the coefficient of uniformity (C_u) but also on the mean grain size (D_{50}). They used a combined grain size index, defined as $I_{GS} = (D_{50}/D_0)/C_u$, where D_{50} is in mm and D_0 is a reference size (1 mm), to account for the influence of grading. Their test data, plotted along with trend lines in Fig. 1c, indicate that ϕ_{cs} increases with I_{GS}. Given the definition of the grain size index, it then follows that for a given D_{50}, the critical state friction angle will increase as the coefficient of uniformity decreases. This result is in contrast with that of Simoni and Houlsby [27] and Kokusho et al. [12]. Since the possible influence of mean grain size was not examined in the two earlier studies, an attempt is made here to reinterpret the test data as a function of I_{GS}, as shown in Fig. 2, where some additional literature data on quartz sands (listed in Table 1) are also included. It is immediately evident from the plot that no clear trend exists that follows the empirical relation of Simonini et al. [28].

While the above-mentioned studies have provided valuable data for understanding the critical state friction angle of granular soils with varying gradation, the contradictory results and views, however, indicate that the influence of grading is much more complex than conventionally thought. Previous studies have often involved different materials (in terms of particle shape, size distribution and mineralogy) and different testing methods, making it difficult to evaluate the discrepancies through direct comparison. In particular, in those studies the influence of particle shape was not isolated from the influence of grading, leading one to speculate that the observed discrepancies might be caused by the influence of

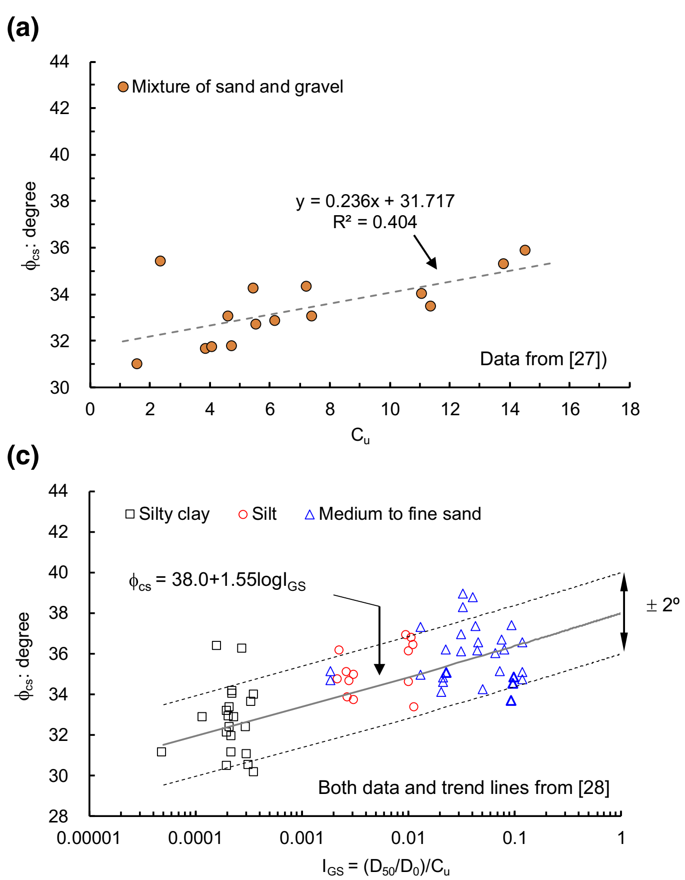

Fig. 1 Observed variation of the angle of friction at critical state with grading

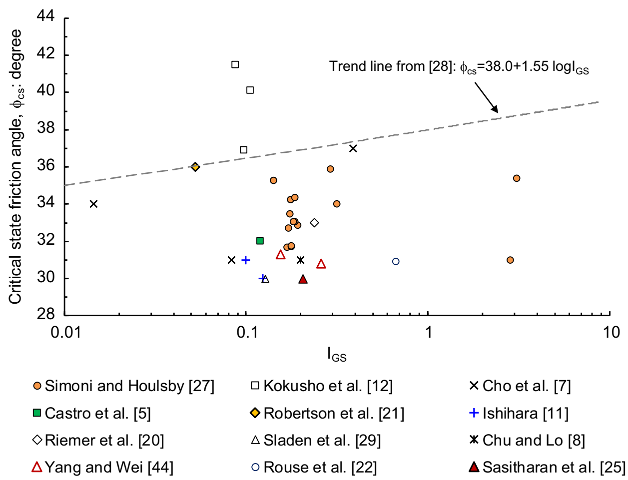

Fig. 2 Literature data showing no clear trend in the variation of critical state friction angle with grain size index

particle shape. It has long been recognized that the angle of friction of granular soils is affected by grain shape (e.g., [7, 14, 31]). More recently, Yang and Luo [43] have provided convincing experimental evidence showing that the critical state friction angle, as well as the position of the critical state locus in the compression space, is sensitive to changes in particle shape and that a qualitative description of particle shape is not adequate. For two uniform granular materials that are both qualitatively classified as rounded, for example, the difference in ϕ_{cs} can be as large as 3°–6°. This difference is comparable to, or even larger than, the range of varying ϕ_{cs} values attributable to the influence of grading in the literature (see Fig. 1).

Given the above observations, a basic question naturally arises: *Does the critical state friction angle of granular material depend on grading?* To address this question is not only of academic interest, but particularly of practical importance since ϕ_{cs} is an important design parameter in many geotechnical applications. If the answer to the question is no, then the current practice of allowing for such influence [4] needs to be taken with caution; otherwise, there is a potential risk of underestimating safety requirements for geotechnical structures. On the other hand, if the effect of grading does exist, then the reasons behind the contradictory results in the literature need to be explored to improve our understanding. This paper presents a study with an attempt to address these concerns through a specifically designed experimental program where the influence of particle shape, mean size and mineralogy was carefully ruled out, with grading being the sole factor varied. Both the angle of friction at critical state and the stress–strain behavior were investigated so as to obtain a comprehensive view. In the following sections, the main results are presented along with detailed interpretation. Several important implications of the results for current geotechnical engineering practice are also provided.

2 Experiments on sand samples of differing gradation

2.1 Particle shape and grading of tested materials

A widely graded quartzitic sand from Fujian, China (referred to as Fujian sand), was used in laboratory testing. Four fractions of the sand with a similar D_{50} were formed by sieving so that the possible influence of mean size was ruled out (Fig. 3). In accordance with the coefficient of

Table 1 Some literature data on critical state friction angles of quartz sands

Material	D_{50}: mm	C_u	I_{GS}	ϕ_{cs}: deg	References
Banding 1 sand	0.18	1.50	0.1200	32.0	Castro et al. [5]
Nevada sand	0.15	1.80	0.0833	31.0	Cho et al. [7]
Ticino sand	0.58	1.50	0.3867	37.0	
Granite powder	0.09	6.20	0.0145	34.0	
Sydney sand	0.30	1.50	0.2000	31.0	Chu and Lo [8]
Kiyosu sand	0.31	2.50	0.1240	30.0	Ishihara [11]
Toyoura sand	0.17	1.70	0.1000	31.0	
Monterey sand	0.38	1.60	0.2375	33.0	Riemer et al. [20]
Brenda sand	0.10	1.90	0.0526	36.0	Robertson et al. [24]
Badger sand	0.87	1.3	0.66923	30.9	Rouse et al. [21]
Ottawa sand	0.35	1.7	0.20588	30.0	Sasitharan et al. [25]
Nerlerk sand	0.23	1.80	0.1278	30.0	Sladen et al. [29]
Fujian sand	0.40	1.53	0.2591	30.8	Yang and Wei [44]
Toyoura sand	0.22	1.39	0.1552	31.3	

$C_u = D_{60}/D_{10}$, $I_{GS} = (D_{50}/D_0)/C_u$

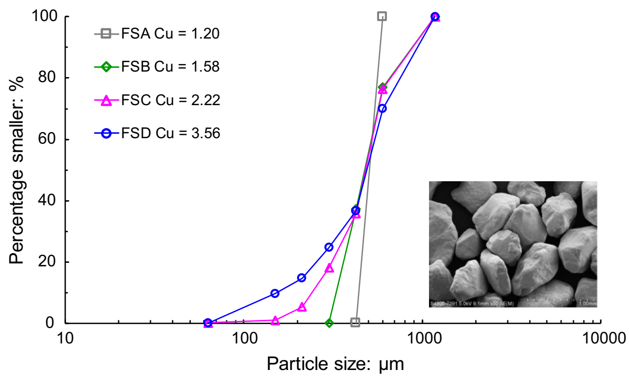

Fig. 3 Particle size distribution curves of four fractions of Fujian sand

uniformity, the four fractions vary from very uniform ($C_u = 1.20$) to moderate grading ($C_u = 3.56$). The overall particle shape of each grading was quantified using the laser scanning technique [30]. For a given particle, three shape measures were made, namely aspect ratio (AR), sphericity (S) and convexity (C). Referring to Fig. 4a–c, the aspect ratio is defined as the ratio between Feret minimum and maximum diameter; the sphericity is defined as the ratio of the perimeter of a circle with the same area as the projected area of the particle to its actual perimeter; and the convexity is the area of the particle (A) divided by its area if any concavities within its perimeter are filled ($A + B$). To obtain representative shape data for each grading, the cumulative distribution curves of AR, S and C were, respectively, established from measurements on tens of thousands of grains, and the values corresponding to the 50% cumulative distribution were selected as the representative AR, S and C values [43].

In addition to the three shape measures, particle roundness, which quantifies the angularity of a particle by the ratio of the average radius of curvature of surface features to the radius of curvature of the maximum inscribed sphere ([35], see Fig. 4d), was also determined for each grading. In so doing, at least 40 grains with clear surface features were picked at random, and the calculated values were then averaged to give the overall roundness (R) for the grading concerned. Table 2 gives a summary of all shape measurements for the four fractions. Additionally, a new shape measure, referred to as overall regularity (OR) and defined as the average of AR, S and C [43], was

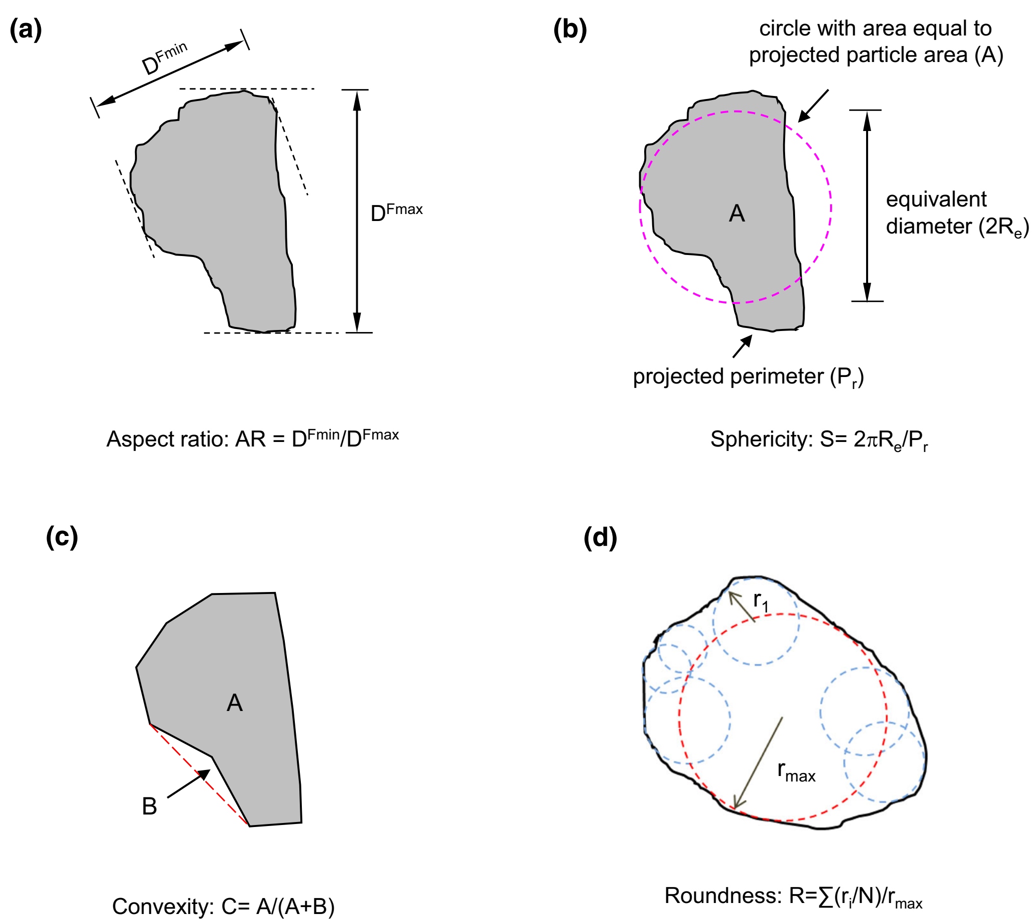

Fig. 4 Schematic illustration of particle shape measures: **a** aspect ratio; **b** sphericity; **c** convexity; **d** roundness

Table 2 Size and shape parameters of Fujian sand of differing gradations

Material	C_u	D_{50}: μm	AR	S	C	R	OR
Fujisan sand A: FSA	1.20	512.5	0.746	0.891	0.956	0.470	0.864
Fujian sand B: FSB	1.58	501.4	0.745	0.891	0.955	0.470	0.864
Fujian sand C: FSC	2.22	501.4	0.745	0.891	0.953	0.470	0.863
Fujian sand D: FSD	3.56	495.6	0.746	0.892	0.951	0.470	0.863

AR = aspect ratio, S = sphericity, C = convexity, R = roundness, OR = overall regularity

determined for each grading to describe its particle shape in a collective manner. As can be seen from Table 2, the four gradations have very similar particle shapes, ensuring that any observed discrepancies in the overall mechanical behavior and in the angle of friction at critical state are attributable to the influence of grading.

2.2 Overall shear behavior at the macroscale

For each grading of Fujian sand, a series of undrained triaxial tests was carried out under a range of states in terms of void ratio and confining stress using an automatic computer-controlled triaxial apparatus. Each specimen was measured 71.1 mm in diameter and 142.2 mm in height and was prepared by the moist tamping method with the under-compaction technique. This method was chosen because it can produce uniform specimens at a wide range of densities. All tests were conducted on saturated specimens. Figure 5 shows the shear responses of four specimens of Fujian sand of Grading B (referred to as FSB; $C_u = 1.58$) in the q-p' and q-ε_a planes. Here q is the deviatoric stress, p' is the mean effective stress, and ε_a is the axial strain in a standard triaxial setting. The initial mean effective stress for the four specimens was 500 kPa, and the basis for comparison was the post-consolidation void ratio. The wide range of void ratios allows observation of a spectrum of shear responses, varying from a dilative, strain-hardening response at dense state ($e = 0.750$) to a highly contractive, liquefaction behavior at very loose state ($e = 0.819$).

In Fig. 6, the shear responses of three specimens of Fujian sand of Grading D (referred to as FSD; $C_u = 3.56$) under the confining stress of 500 kPa are presented. Clearly, all the three specimens were highly contractive under loading although their void ratios were all substantially lower than that of the densest specimen of Grading B in Fig. 5. This implies that the difference in grading can impose a marked impact on the overall shear behavior. To make the point clear, two specimens, one formed from Fujian sand of Grading A (FSA; $C_u = 1.20$) and the other formed from Fujian sand of Grading C (FSC; $C_u = 2.22$), were carefully controlled to arrive at almost the same post-consolidation state ($e = 0.729$ and $p' = 500$ kPa) and then sheared under the undrained condition. The results are

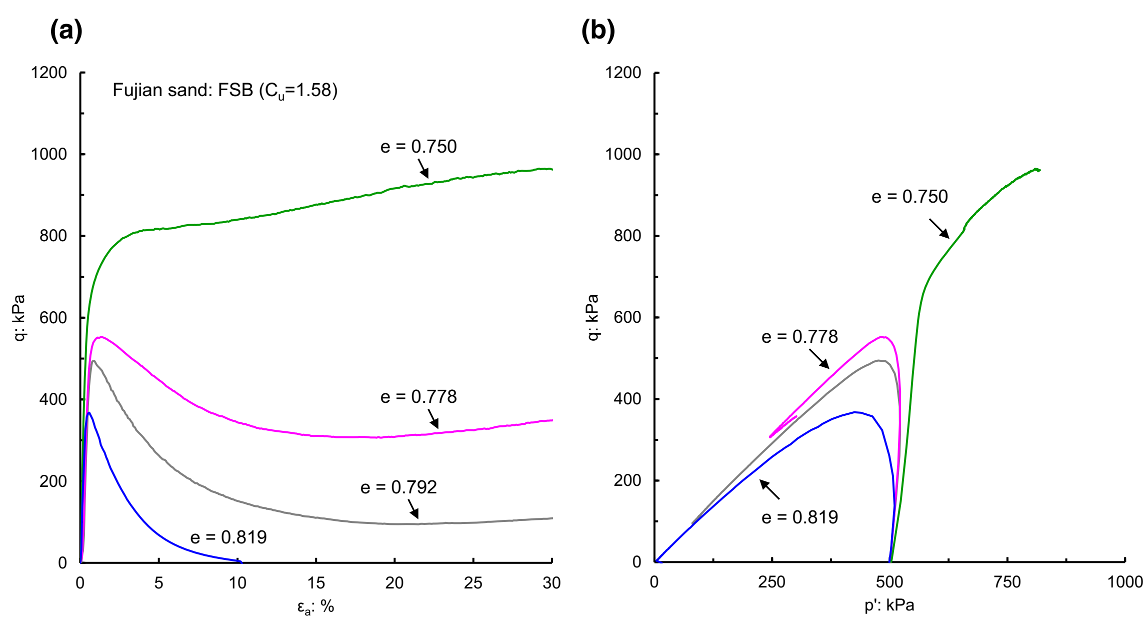

Fig. 5 State-dependent shear behavior of Fujian sand FSB under undrained condition: **a** stress–strain relation; **b** stress path

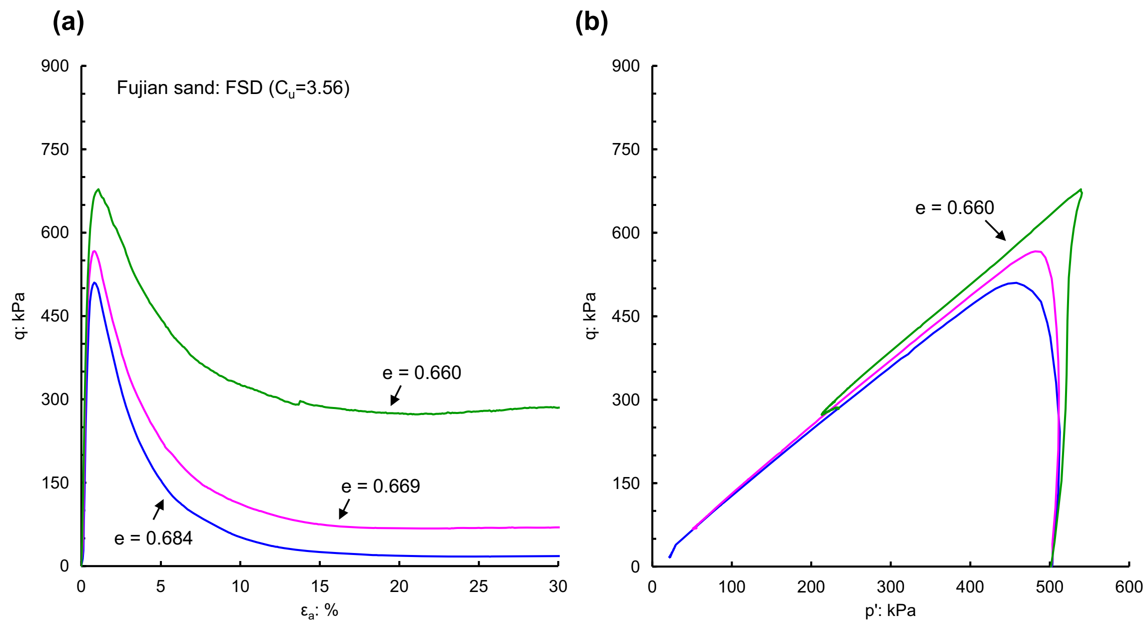

Fig. 6 State-dependent shear behavior of Fujian sand FSD under undrained condition: **a** stress–strain relation; b stress path

compared in Fig. 7. It is interesting to note that the specimen of uniform sand FSA exhibited a strongly dilative response and achieved a much high strength at large strains, whereas the specimen of less uniform sand FSC underwent liquefaction with almost complete loss of strength at large strains.

2.3 Influence of grading on critical state friction angle

Of more interest here is to examine what the influence of grading is on the angle of friction at critical state. In doing so, the critical state data derived from the tests are plotted in q-p' plane for the four gradations, as shown in Fig. 8. Apparently, all data points can be perfectly fitted by a single line passing through the origin, yielding the critical stress ratio M of 1.21. This value corresponds to the angle of friction ϕ_{cs} of 30.2°, which is typical for uniform quartz sands as reported in the literature (see Table 1). The results in Fig. 8 provide solid evidence that *under otherwise identical conditions, the critical state friction angle is not, as commonly thought, affected by particle grading*.

3 Experiments on samples of varying particle shape

3.1 Strategy for varying particle shape

As no influence of grading is observed on ϕ_{cs}, the reason for the literature reported variation of ϕ_{cs} with grading needs to be sought. We speculate that this variation is mainly associated with differing particle shapes of the granular soils tested. However, in the previous studies particle shapes of the tested soils were either not quantified or only described in qualitative terms. To verify the hypothesis, additional tests on granular materials with an identical grading but varying particle shapes were devised. In doing so, a sequence of binary mixtures were formed by mixing Fujian sand of Grading A ($C_u = 1.20$), respectively, with as-supplied and crushed glass beads of the *same* grading. The as-supplied glass beads were highly spherical, whereas the crushed glass beads were very angular. Following the concept of combined shape parameter for binary mixtures [44], the overall particle shape of so-formed mixtures was systematically varied by varying the percentage of as-supplied or crushed glass beads. For example, for a binary mixture formed by Fijian sand of 60% and glass beads of 40% (denoted as FS60G40; percentage by mass), its combined aspect ratio is determined by

$$AR_{\mathrm{FS60G40}} = AR_{\mathrm{Fujiansand}} \times 60\% + AR_{\mathrm{glassbeads}} \times 40\%. \quad (2)$$

The above expression gives an AR value of 0.837 for the binary mixture, which is quite reasonable compared with 0.746 for pure Fujian sand of Grading A and 0.974 for pure glass beads. Similarly, the combined aspect ratio for binary mixture FS80C20, formed by mixing 80% Fujian sand and 20% crushed glass beads, can be determined as 0.727 compared with 0.653 for pure, angular crushed glass beads. All shape data for the binary mixtures tested are summarized in Table 3.

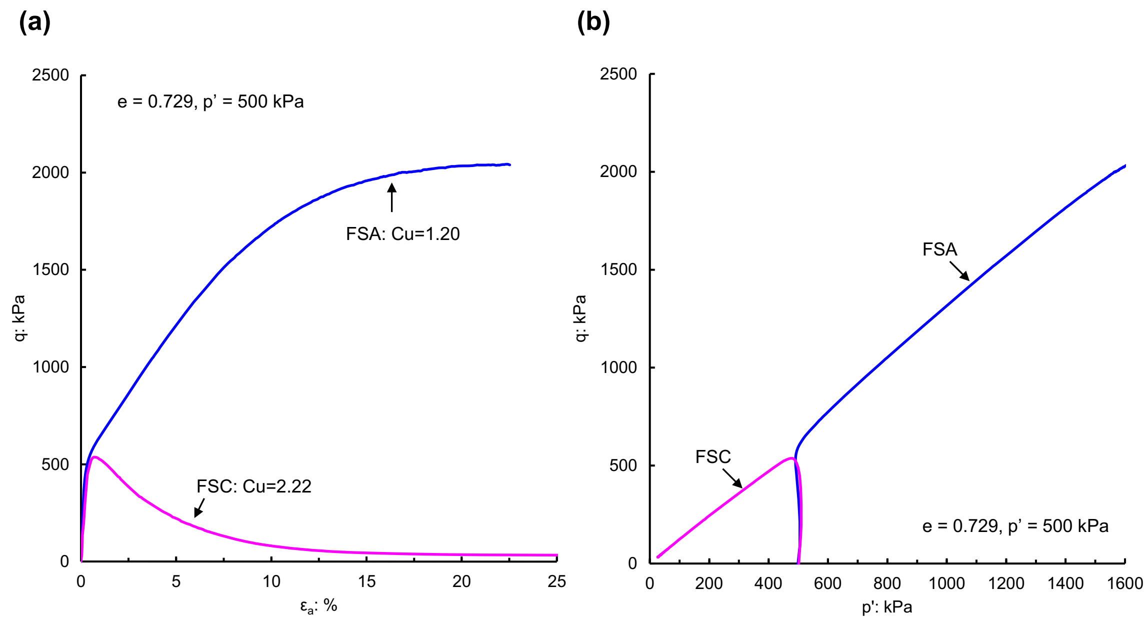

Fig. 7 Influence of grading on undrained shear behavior of Fujian sand (FSA versus FSC): **a** stress–strain relation; **b** stress path

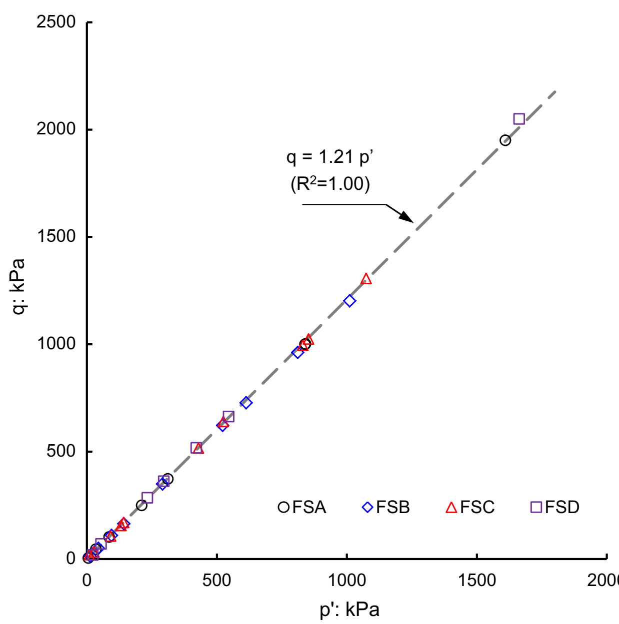

Fig. 8 Locus of critical states in stress space

3.2 Overall shear behavior at the macroscale

The undrained shear responses of mixtures FS60G40 and FS60C40 are compared in Fig. 9a, b. Referring to Table 3, the overall regularity and roundness of FS60G40 are 0.904 and 0.682 compared with 0.839 and 0.438 for FS60C40. This means the former mixture contained particles that were more regular and rounded than the latter. The two specimens were carefully controlled to achieve the same post-consolidation state ($e = 0.820$, $p' = 500$ kPa) in the compression space so as to afford a convincing comparison. It is evident that the two specimens responded in distinctly different manners. The behavior of the mixture FS60C40 was dilative and strain hardening, whereas the mixture FS60G40 completely liquefied. Similar results were obtained from tests on mixtures FS80G20 and FS80C20, as shown in Fig. 9c, d. Given the identical grading of these mixtures, it is convincing to attribute the marked differences in the overall behavior to the differences in their particle shapes.

In addition to the undrained tests, some drained tests were also performed and the effect of particle shape on the overall strain–strain behavior was observed as well. Given the scope of this paper, these results are not presented here except for the critical state data that will be discussed in the following section.

3.3 Influence of particle shape on critical state friction angle

For comparison, the critical state data for the pair of mixtures FS60G40 and FS60C40 are plotted in Fig. 10a, while the data for another pair of mixtures FS80G20 and FS80C20 are plotted in Fig. 10b. For each mixture, both undrained (CU) and drained (CD) test data can be well fitted by a straight line passing through the origin (meaning that the critical state friction angle is independent of drainage conditions), but the slopes of these best-fit lines differ from each other and all differ from that of pure Fujian sand shown in Fig. 8. For example, the M value for mixture FS60G40 is quantified as 1.08, whereas the M value for

Table 3 Size and shape parameters of binary mixtures

Material	C_u	D_{50}: μm	AR	*S*	*C*	*R*	OR
FS80G20	1.20	512.5	0.791	0.901	0.960	0.576	0.884
FS60G40	1.20	512.5	0.837	0.912	0.963	0.682	0.904
FS80C20	1.20	512.5	0.727	0.877	0.951	0.454	0.852
FS60C40	1.20	512.5	0.709	0.863	0.945	0.438	0.839
Glass beads	1.20	512.5	0.974	0.944	0.974	1.000	0.964
Crushed glass beads	1.20	512.5	0.653	0.822	0.929	0.390	0.801

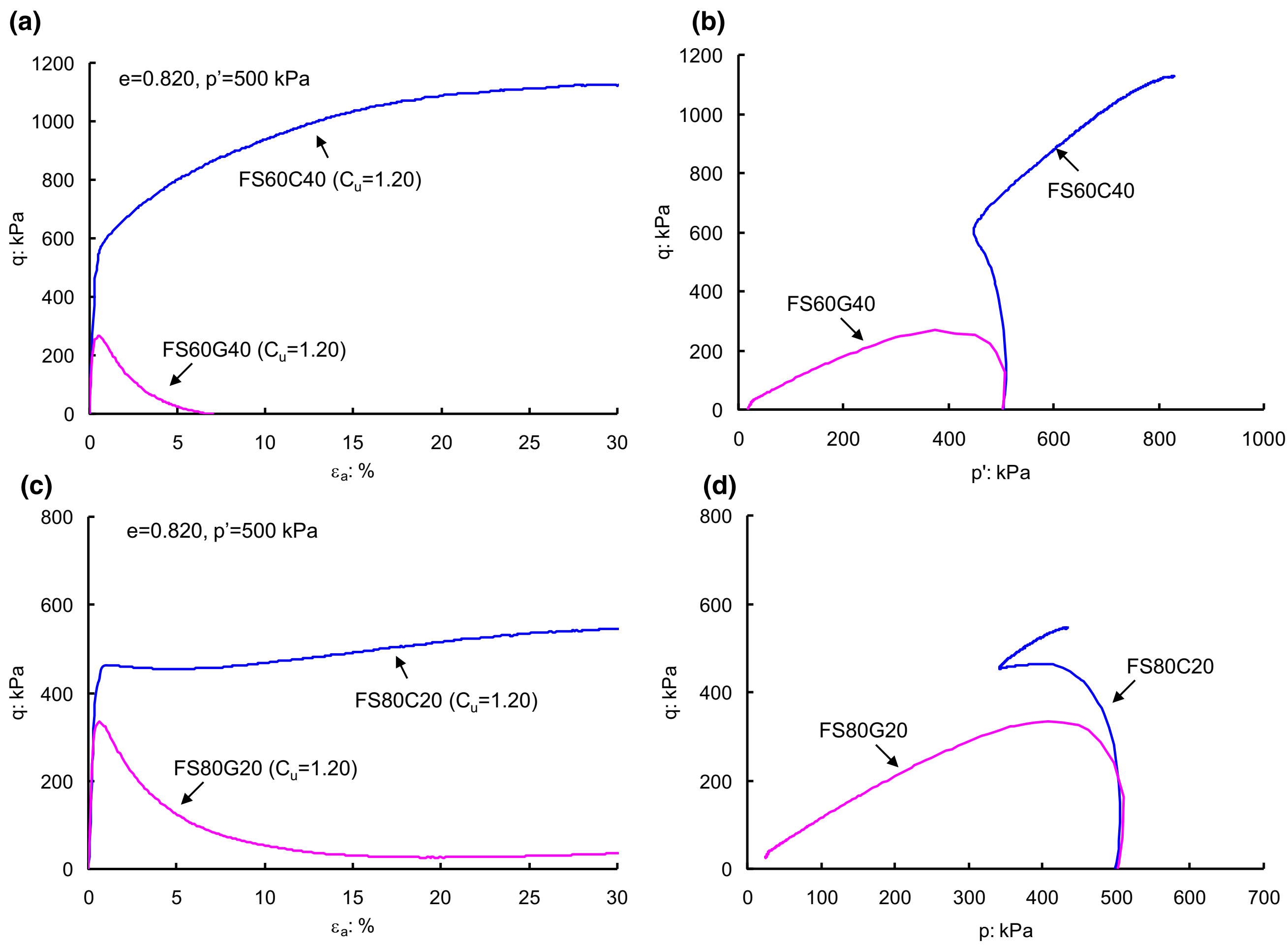

Fig. 9 Influence of grain shape on shear response of binary mixtures with identical grading: **a** and **c** stress–strain relation; **b** and **d** stress path

FS60C40 is 1.34. Since FS60G40 and FS60C40 share the same grading and same mineral compositions, the difference in *M* values is believed to be mainly associated with the difference in their particle shapes.

Figure 11 shows how the value of ϕ_{cs} for Fujian sand varies with the percentage of as-supplied or crushed glass beads. As discussed before, this variation is thought to be linked with varying particle shape that is reflected by the percentage of the additives through the combined shape parameter defined in Eq. (2). A notable feature of Fig. 11 is that ϕ_{cs} tends to *increase* as the percentage of crushed glass beads is increased, but it tends to *decrease* with increasing the percentage of glass beads. This finding is in general agreement with the experimental results on the effect of adding non-plastic fines to clean sand [36, 44]. It is noteworthy, however, that all materials in this study are of sand size with uniform grading (C_u = 1.20) rather than of gap grading in the case of sand–fines mixtures. In this respect,

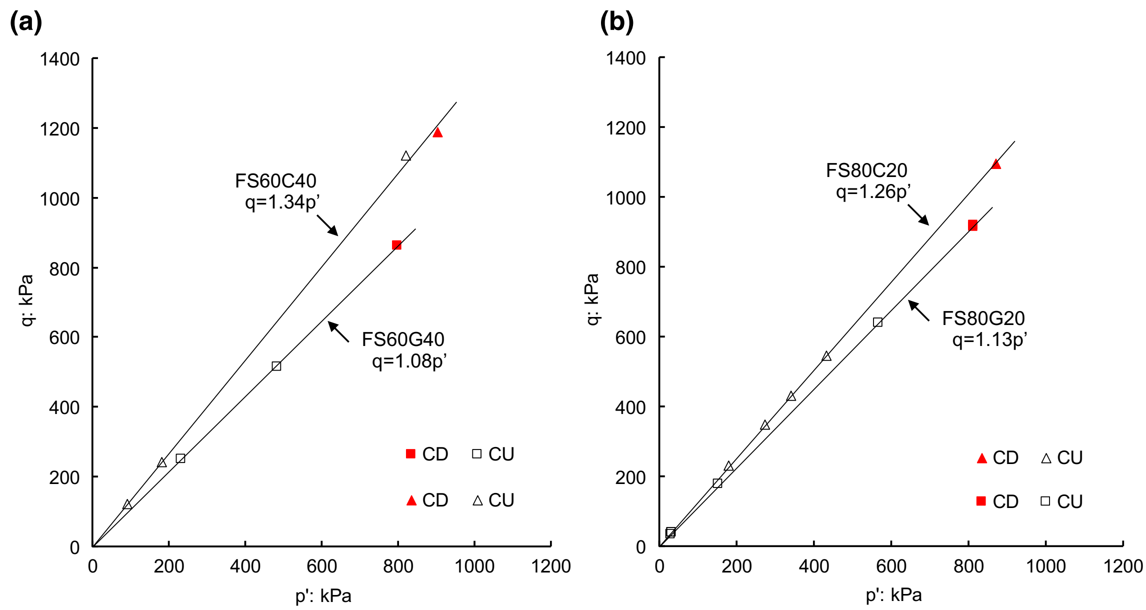

Fig. 10 Loci of critical states of binary mixtures with identical grading in stress space: **a** samples of FS60C40 and FS60G40; **b** samples of FS80C20 and FS80G20

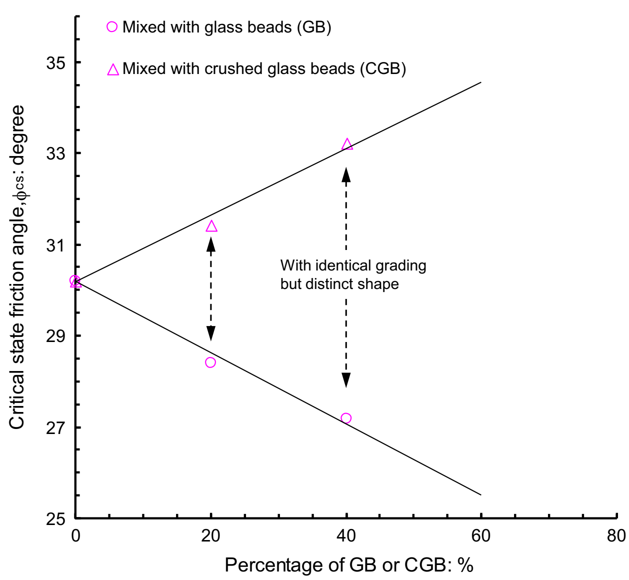

Fig. 11 Variation of critical state friction angle with percentage of *glass beads* and *crushed glass beads*

the agreement reinforces the proposition that the variation of ϕ_{cs} with the addition of non-plastic fines into clean sand is mainly associated with the change of particle shape rather than the change of grading. It also suggests that the observed variation of critical state friction angle in shear tests where a significant particle crushing occurred (e.g., [22]) is most likely attributable to *changing particle shape* caused by particle breakage rather than to changing gradation. This proposition is also supported by DEM simulations on the effect of particle shape where particle breakage was not allowed (e.g., [15, 39]).

To quantify the influence of particle shape on critical state friction angle, Fig. 12a shows ϕ_{cs} as a function of overall regularity (OR) and Fig. 12b shows ϕ_{cs} as a function of roundness (R). A fairly good correlation is observed in each plot, which can be approximately described as follows

$$\phi_{cs} = 109.66 - 91.58\mathrm{OR}, \quad (3)$$

$$\phi_{cs} = 41.38 - 21.5R. \quad (4)$$

where the angle of friction is in degree. The trend given by the above correlations is generally consistent with the results from several previous studies on natural and crushed sands that ϕ_{cs} decreases with particle roundness (e.g., [7, 21]). For example, the correlation between ϕ_{cs} and R proposed by Cho et al. [7] is as follows:

$$\phi_{cs} = 42 - 17R. \quad (5)$$

While the trend is similar, it is worth noting that the relation in Eq. (4) is established on the basis of systematic tests where the influence of grading was isolated and the particle shape was accurately measured. In this respect, Eq. (4) is anticipated to provide more accurate and rational predictions. This is demonstrated by examining the limiting case of $R = 1$, which represents granular assemblies of perfectly spherical particles. The relation in Eq. (5) gives ϕ_{cs} a value of 25°, whereas the relation proposed in this study gives a value of about 20°. Note that the smaller value is quite close to the critical state friction angle of

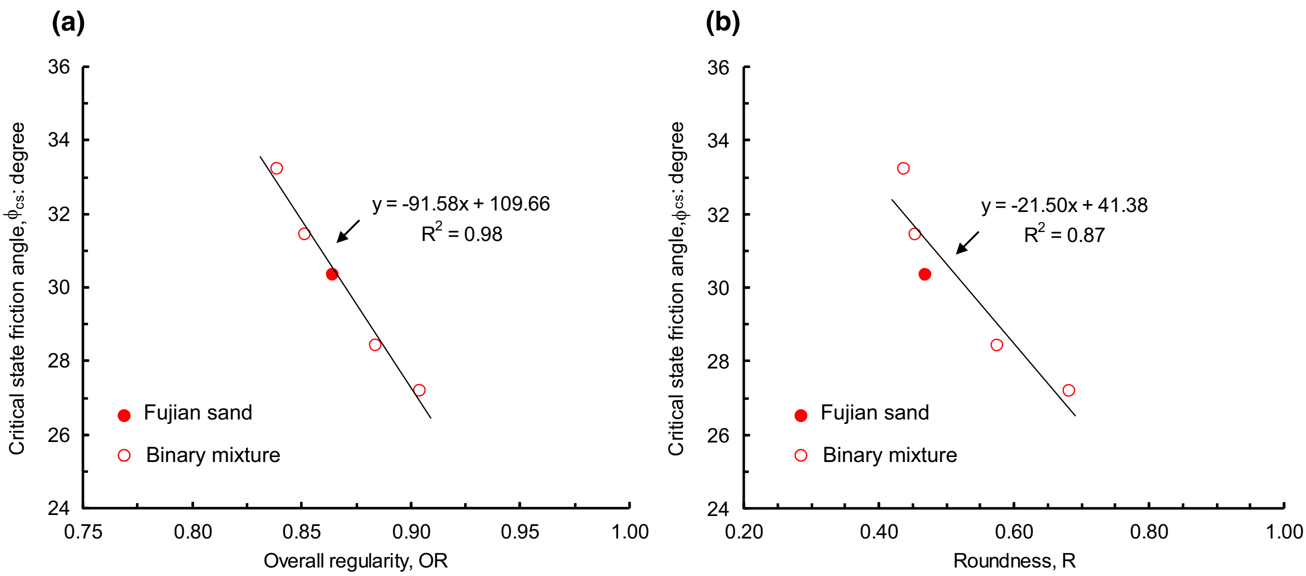

Fig. 12 Correlation of critical state friction angle with **a** particle regularity and **b** particle roundness

spherical glass beads determined by Cavarretta et al. [6] using triaxial tests (~21°).

4 Implications for engineering practice

Based on the specifically designed experiments, several important aspects of the critical state friction angle have been identified. Further examination of the implications of these findings for current engineering practice is worthwhile.

4.1 Implications for drained stability problems

The critical state friction angle of a granular soil is generally assumed as a constant independent of initial density and of drainage conditions. It corresponds to the minimum drained strength of a soil and hence informs the safety requirement in geotechnical design. In the *Code of Practice for Earth Retaining Structures* [4], it is recommended that for siliceous sands and gravels ϕ_{cs} may be estimated as follows

$$\phi_{cs} = 30 + \Delta\phi_1 + \Delta\phi_2, \qquad (6)$$

where $\Delta\phi_1$ is the contribution to ϕ_{cs} from the angularity of particles and $\Delta\phi_2$ is the contribution to ϕ_{cs} from the soil's particle size distribution. As documented in Table 4, the values of $\Delta\phi_1$ are assumed to be 0 for rounded to well-rounded soils, 2 for subangular to subrounded soils and 4 for very angular to angular soils, whereas the values of $\Delta\phi_2$ are taken in accordance with the soil's uniformity coefficient: For $C_u < 2$, $\Delta\phi_2$ is 0, for $2 \leq C_u < 6$, $\Delta\phi_2$ is 2, and for $C_u \geq 6$, $\Delta\phi_2$ is 4.

Evidently, the Code of Practice assumes that the critical state friction angle is dependent on both particle shape and grading and the effects arising from these two factors are allowed for *separately*. This guidance is, however, not supported by the laboratory test results shown in Fig. 8 where the two factors were carefully controlled. It is not supported by the literature data reinterpreted in Fig. 2 either. In this respect, the guidance *is not considered appropriate*. The effect of grading reported in the literature is very likely attributable to varying particle shape, as evidenced by the experimental data in Figs. 10, 11 and 12 where all tested materials were controlled to have the same grading but different particle shapes. Note that the assumed contribution to ϕ_{cs} from the effect of grading in [4] is as large as 4°. The extra strength arising from this difference is not insignificant, but rather can lead to different safety requirements in geotechnical design (e.g., [3]) and hence different costs. In this connection, the practice of estimating ϕ_{cs} in accordance with Eq. (5) should be taken with caution. Wherever possible and practical, it is recommended that the value of ϕ_{cs} be measured.

In addition, it should be noted that in allowing for the influence of particle shape in Eq. (6), the particle shape of a given soil is described in qualitative terms on the basis of some empirical charts (e.g., [13, 19]). Due to the subjective nature of the visual description, two granular soils with different shape parameters may be classified into the same category, thus leading to the same $\Delta\phi_1$ value. However, the experimental results shown in Fig. 12 indicate that ϕ_{cs} is sensitive to changes in shape parameter and the visual description may not be adequate. Particularly, for granular materials with highly rounded particles ϕ_{cs} can take values that are substantially smaller than the lower bound

Table 4 Recommended practice for estimating critical state friction angle (after BSI [4])

Soil property	Determined from	Classification	Parameter
Angularity of particles	Visual description	Rounded to well rounded	$\Delta\phi_1 = 0$
		Subangular to subrounded	$\Delta\phi_1 = 2$
		Very angular to angular	$\Delta\phi_1 = 4$
Coefficient of uniformity, C_u	Soil grading	$C_u < 2$	$\Delta\phi_2 = 0$
		$2 \le C_u < 6$	$\Delta\phi_2 = 2$
		$C_u \ge 6$	$\Delta\phi_2 = 4$

$\phi_{cs} = 30 + \Delta\phi_1 + \Delta\phi_2$

specified by Eq. (6). This calls for attention to the potential risk of overestimating safety in geotechnical design.

4.2 Implication for liquefaction susceptibility assessment

In the design of large-scale earth structures such as hydraulic fills for artificial islands or tailings dams, a major concern is the susceptibility of the granular materials to flow failure or liquefaction [10, 29]. The concept of critical state or steady state has been widely used as a basis for liquefaction analysis [11, 18, 33]. Therefore, it is of considerable interest to examine the implications of the experimental findings in this respect.

For a strain softening response in q-p' space, the straight line passing through the origin and the peak point (i.e., instability state) on the stress path is useful in characterizing the onset of flow liquefaction (see the inset of Fig. 13). Yang [41] has proposed that the slope of this flow liquefaction line is not unique, but depends on the state of the soil in terms of a state parameter defined with reference to the critical state locus as follows

$$\left(\frac{q}{p'}\right)_{\text{atpeakpoint}} = \frac{M}{B}\exp(A\Psi), \tag{7}$$

where the state parameter Ψ measures the difference between the current void ratio and the critical void ratio at the current mean effective stress [1]; M is the stress ratio at the critical state; and A and B are two parameters. If specimens of a soil are sheared from the same stress level, then the slope of the flow liquefaction line is expected to be an exponential function of the void ratio. This inference is indeed supported by the data in Fig. 13, where values of the stress ratio at the peak state for specimens of Fujian sand sheared from $p' = 500$ kPa are plotted against the void ratio. For a given grading, the stress ratio tends to decrease with increasing void ratio, meaning that the liquefaction susceptibility will become higher as the soil becomes looser—this is certainly a sound trend.

Of more interest is the observation that the trend curve tends to shift to the left as the coefficient of uniformity

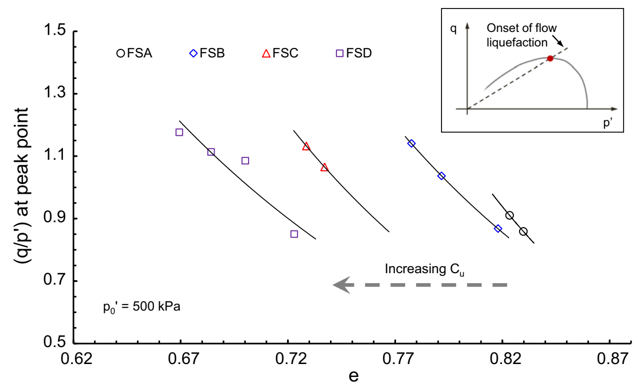

Fig. 13 Variation of flow *liquefaction line* with state parameter

increases. This yields an important implication that *at a given void ratio, the stress ratio to trigger flow liquefaction for a well-graded soil would be lower than that for a uniformly graded soil*. While it is somehow surprising at first glance, the influence of grading on liquefaction susceptibility has been found explainable in the framework of critical state theory. Figure 14 shows the critical state loci (CSL) for the four gradations of Fujian sand in the e-logp' plane. To determine critical states with good confidence, laboratory tests on loose, contractive specimens under undrained conditions are preferred, and the tests with a significant strain localization should not be used. An evident feature of Fig. 14 is that the CSL undergoes a downward shift as the coefficient of uniformity increases. A similar effect of changing gradation on the position of CSL was also observed in several DEM simulations (e.g., [15, 16, 38, 40]). In addition, Fig. 14 shows that the CSL is a curved rather than straight line in the e-logp' plane. This feature has also been observed in laboratory studies on different sands (e.g., [34, 44]). The curvature of CSLs is not considered as the consequence of particle breakage because the mean stress level involved in the tests was much lower than that required to break sand particles. This proposition is confirmed by comparing particle size distribution curves before and after testing for selected samples, as shown in Fig. 15 for FSC.

The variation in position of the CSL implies that at a given confining pressure and void ratio, a well-graded

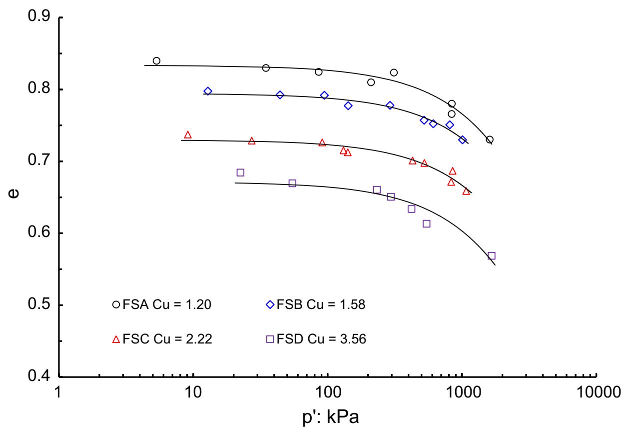

Fig. 14 Influence of grading on the position of critical state locus in compression space

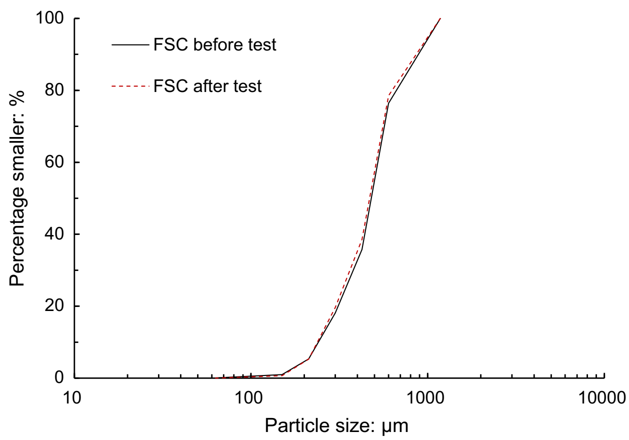

Fig. 15 Particle size distribution curves of FSC before and after testing

material would be at a looser state and hence more susceptible to liquefaction than a uniformly graded material. Note that the observed movement of the CSL with varying gradation (C_u) in the compression space is, to some extent, in analogy to the observation on sand–fines mixtures that the CSL tends to move down with increasing fines content [32, 44]. In this connection, one may conclude that a clean sand with a certain amount of fines (e.g., 10% by mass) tends to be more susceptible to liquefaction than the clean sand itself when compared at the same post-consolidation state in terms of void ratio and confining pressure.

5 Summary and conclusions

This paper has raised a question that is of considerable interest from both theoretical and practical perspectives: *Whether the critical state friction angle of granular materials is dependent on grading*. The well-designed experiments have produced comprehensive data sets that help to resolve the question. The significant findings are summarized as follows.

(1) The current literature contains diverse and even contradictory views on the effect of grading. The observations reported in the literature that the critical state friction angle either *increases or decreases* with the coefficient of uniformity are most likely attributable to varying particle shapes of the tested materials, rather than to varying particle size distributions.

(2) The laboratory tests on four gradations of a quartz sand where the influence of particle shape was ruled out provide solid evidence that the angle of friction at critical state is not affected by grading, whereas the systematic tests on a sequence of binary mixtures with an identical grading but different grain shapes show that the critical state friction angle is highly dependent on particle shape.

(3) The current practice of allowing for the effect of grading on critical state friction angle should be taken with caution or be discontinued to avoid the risk of underestimating safety requirements in geotechnical design. Caution should also be taken about the inadequacy of using the visual description to allow for the influence of particle shape, because the critical state friction angle is sensitive to changes in shape parameter. Wherever possible, the critical state friction angle should be measured.

(4) Although varying gradation has no influence on the angle of friction at critical state, it can impose a marked impact on the liquefaction susceptibility of granular soils. Under similar post-consolidation conditions, a well-graded material would be more susceptible to liquefaction than a uniformly graded material. Care should be exercised about the two distinct effects arising from particle grading in the design of hydraulically placed fills and tailings dams.

(5) The variation of liquefaction susceptibility caused by varying gradation is shown to be consistent with the variation in location of the critical state locus (CSL) in the compression space: As the coefficient of uniformity increases, the CSL tends to move downward. This consistency suggests that the critical state theory can provide a coherent explanation for the observed effect.

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