

# Frequency-Dependent Amplification of Unsaturated Surface Soil Layer

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**Abstract:** This paper presents a study of the amplification of SV waves obliquely incident on a surface soil layer overlying rock formation. Special attention is placed on the influence of the saturation states of the soil layer and the bedrock on the amplification in both horizontal and vertical directions as well as on the amplitude ratios between the two directions at the surface, where the vertical and horizontal amplification and the amplitude ratios are expressed as functions of the frequency of incident waves. The analysis indicates that while the influence of the saturation state of the bedrock is insignificant, a change of the saturation state of the soil layer may have a marked impact on the vertical amplification. For typical seismic frequencies, an unsaturated soil layer can generate greater vertical amplification than a saturated layer; it can also cause larger amplitude ratios between vertical and horizontal components at the surface. The analysis further confirms the potential importance of the saturation condition of near-surface soils in site response analysis.

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## Introduction

The ground motion induced by an earthquake is in both horizontal and vertical directions. Current studies of earthquake effects have tended to disregard the vertical ground motion and concentrated mainly on the horizontal ground motion. If the effect of vertical motion is explicitly included in design, it is typically assumed that the ratio of vertical to horizontal ( $V/H$ ) response spectra will not exceed two-thirds (e.g., ICC 1997). However, observations from recent earthquakes, such as the 1994 Northridge and 1995 Kobe earthquakes (NCEER 1997), have indicated that the rule-of-thumb ratio of two-thirds is a poor descriptor of vertical ground motion. The ( $V/H$ ) spectral ratios may substantially exceed two-thirds in the near field of moderate and large earthquakes and for short periods, and they may be significantly influenced by local soil conditions.

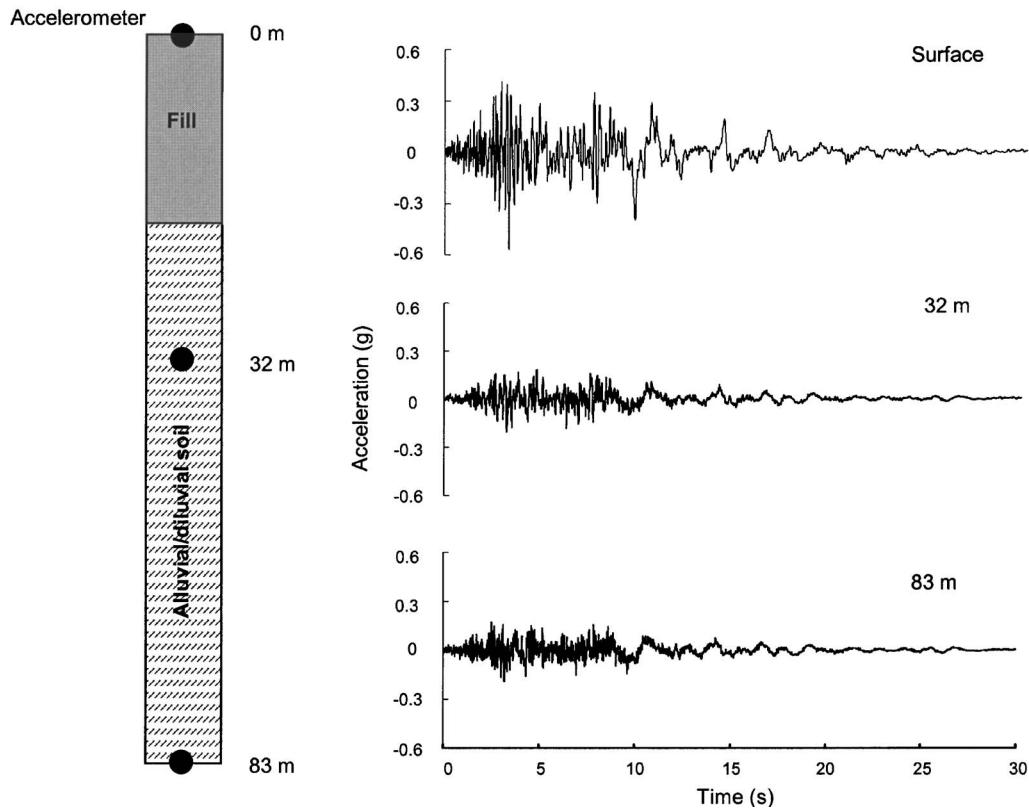
There is excellent evidence coming from a borehole array site subjected to the Kobe earthquake, which showed that the peak vertical acceleration was twice as high as the peak horizontal acceleration at ground surface. A detailed analysis of the three-dimensional borehole array records (Yang and Sato 2000) indicated that the partial saturation condition of near-surface soils played a key role in the vertical motion amplification (see Fig. 1). This analysis has stimulated great interest in the effect of partial saturation on seismic ground motion and seismic soil-structure interaction (Mylonakis and Gazetas 2002; Wang and Hao 2002; Mucciarelli et al. 2003; Lin et al. 2005).

In geotechnical analyses, it is customary to assume that the amplification of vertical ground motion relates to vertically propagating P waves while the amplification of horizontal ground motion is associated with vertical SH waves. This classical assumption in soil amplification is very simplified; in a real seismic environment, the amplification in either horizontal or vertical direction may also be due to inclined SV-P or surface waves. Aimed at clarifying several basic issues on the effect of saturation, Yang (2002) studied the response of a semi-infinite homogeneous soil medium induced by obliquely incident SV-waves, where the soil was modeled as a porous material either fully or partially saturated. The results indicate that even a slight decrease of full saturation in the soil may cause a substantial influence on the surface amplitudes in both horizontal and vertical directions as well as the amplitude ratios between them. It is also found that a significant phase shift of the particle motion in the horizontal and vertical components may occur due to a slight change in saturation.

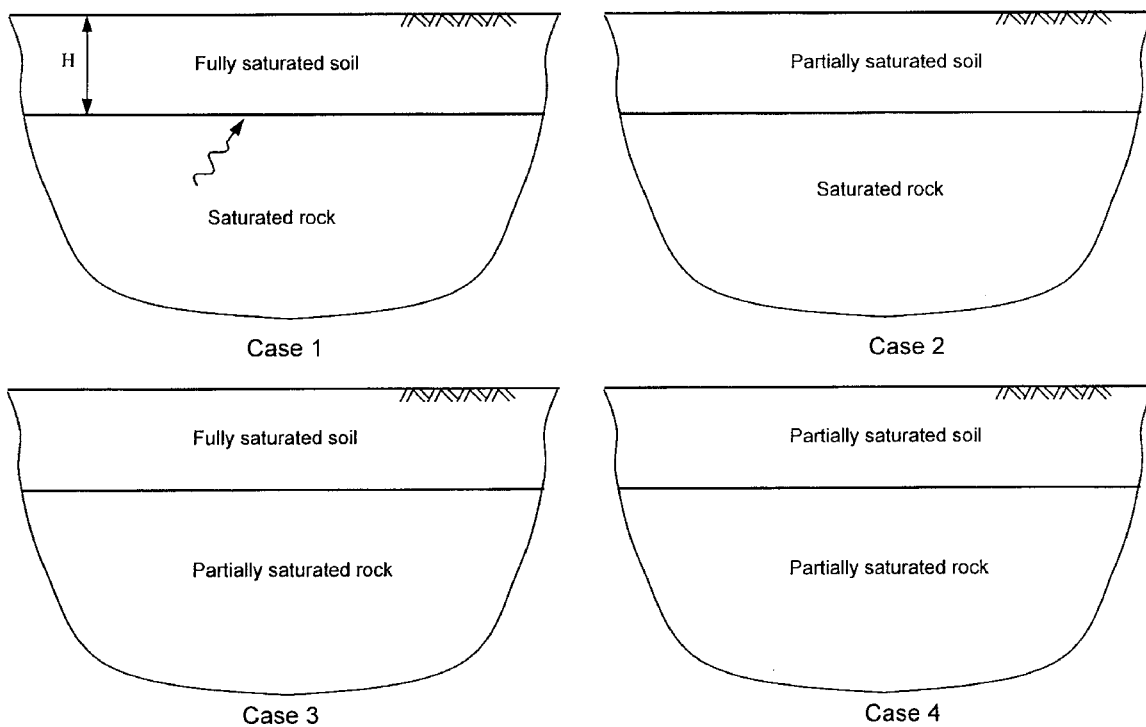
A model of more practical interest is a surface soil layer of finite thickness overlying infinite rock formation. This one-layer model has been extensively discussed in the past by assuming the soil and bedrock as elastic or viscoelastic solids (e.g., Chen et al. 1981; Wolf 1985). Because of the existence of groundwater, both the surface soil and the underlying rock can be either saturated or unsaturated. There are, in total, four possible cases of saturation, as illustrated in Fig. 2. Yang (2001) developed an analytical formulation for computation of the ground motion induced by inclined SV waves in such a layered system, and presented some preliminary numerical results. Due to the difficulty in numerical computation in the early stage, all of the results were obtained only for the displacement amplitudes at the surface, not the ratios between the amplitudes at the surface and at the base of the layer; and all of the results were for a single frequency. Moreover, the earlier analysis assumed the bedrock to be fully saturated, without considering the case of unsaturated bedrock.

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**Fig. 1.** Vertical ground motion recorded by a borehole array during the Kobe earthquake



**Fig. 2.** Four cases analyzed

**Table 1.** Properties of Sand and Rock Used in Analysis

	$G_s$	$n$	$G$ (MPa)	$K_b$ (MPa)	$K_s$ (MPa)	$k$ (m/s)
Sand	2.65	0.45	40	66.7	36,000	$10^{-4}$
Bedrock	2.61	0.32	900	1,500	36,000	$10^{-7}$

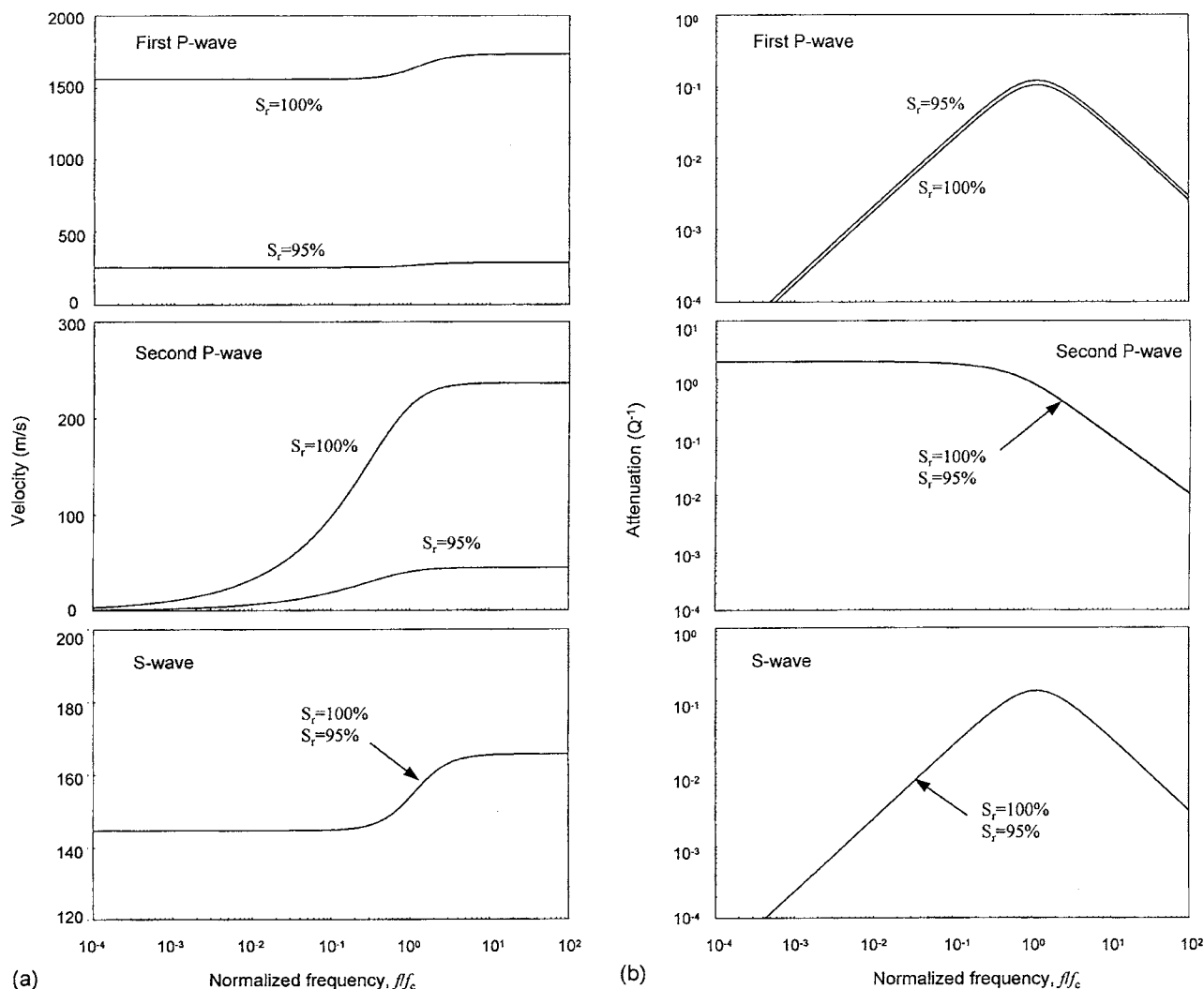
Note:  $G_s$ =specific gravity;  $n$ =porosity;  $G$ =shear modulus;  $K_b$ =bulk modulus of solid skeleton;  $K_s$ =bulk modulus of solid grains; and  $k$ =permeability.

It is of more practical value, however, to estimate the frequency-dependent amplification in site response analysis. There is also a great interest to clarify whether the saturation state of the underlying bedrock has an influence on the amplification. It is the purpose of the present work to identify how the saturation states of the surface soil and the bedrock affect the frequency-dependent amplification, in both horizontal and vertical directions, and particularly, how the saturation states affect the frequency-dependent amplitude ratios between the two components at the surface. The understanding gained from the study may provide more useful implications for the site evaluation

technique, known as ( $H/V$ ) spectral ratio method (Nakamura 1989). The method involves interpreting the surface horizontal-to-vertical spectral ratios of microtremors and has become increasingly popular in recent years (Mucciarelli et al. 2003; Arai and Tokimatsu 2004).

### Effect of Saturation on Propagation of P and S Waves

For the first instance, consider a 30 m sand layer above rock formation. Both the sand and bedrock are regarded as a porous material that is characterized by its porosity,  $n$ , degree of saturation,  $S_r$ , permeability,  $k$ , and the compressibility of solid and fluid constituents. The properties of the sand and bedrock are given in Table 1; they respectively represent typical loose sand and soft rock (JSCE 1994). The formulation of analysis for the single-layered model can readily be developed following the general framework presented by Yang (2001) for a multilayered system. For the sake of conciseness, the mathematical formulation is not described herein.



**Fig. 3.** Frequency-dependent (a) velocities and (b) attenuations of P and S waves in sand

**Table 2.** Degrees of Saturation Used in Four Cases

	Case 1	Case 2	Case 3	Case 4
Sand ( $S_r$ )	100%	95%	100%	95%
Bedrock ( $S_r$ )	100%	100%	95%	95%

Note:  $S_r$ =Degree of saturation.

Unlike the simple case of vertically propagating waves, the obliquely incident SV waves involve both compressional and shear motions, and thereby induce both horizontal and vertical displacements in the soil and the bedrock. More complicatedly, a fluid-saturated material can sustain three body waves: Two P waves and one shear wave (Biot 1956). All three waves are dispersive and dissipative, namely, the wave propagation is frequency dependent and attenuated. Using the properties of the sand given in Table 1, the values of the velocity and attenuation ( $Q^{-1}$  values) of the three waves are computed as a function of frequency and shown in Fig. 3. Here, the frequency  $f$  is normalized by a so-called characteristic frequency for the sand, defined as  $f_c = ng/2\pi k$ , where  $g$  is the gravitational acceleration.

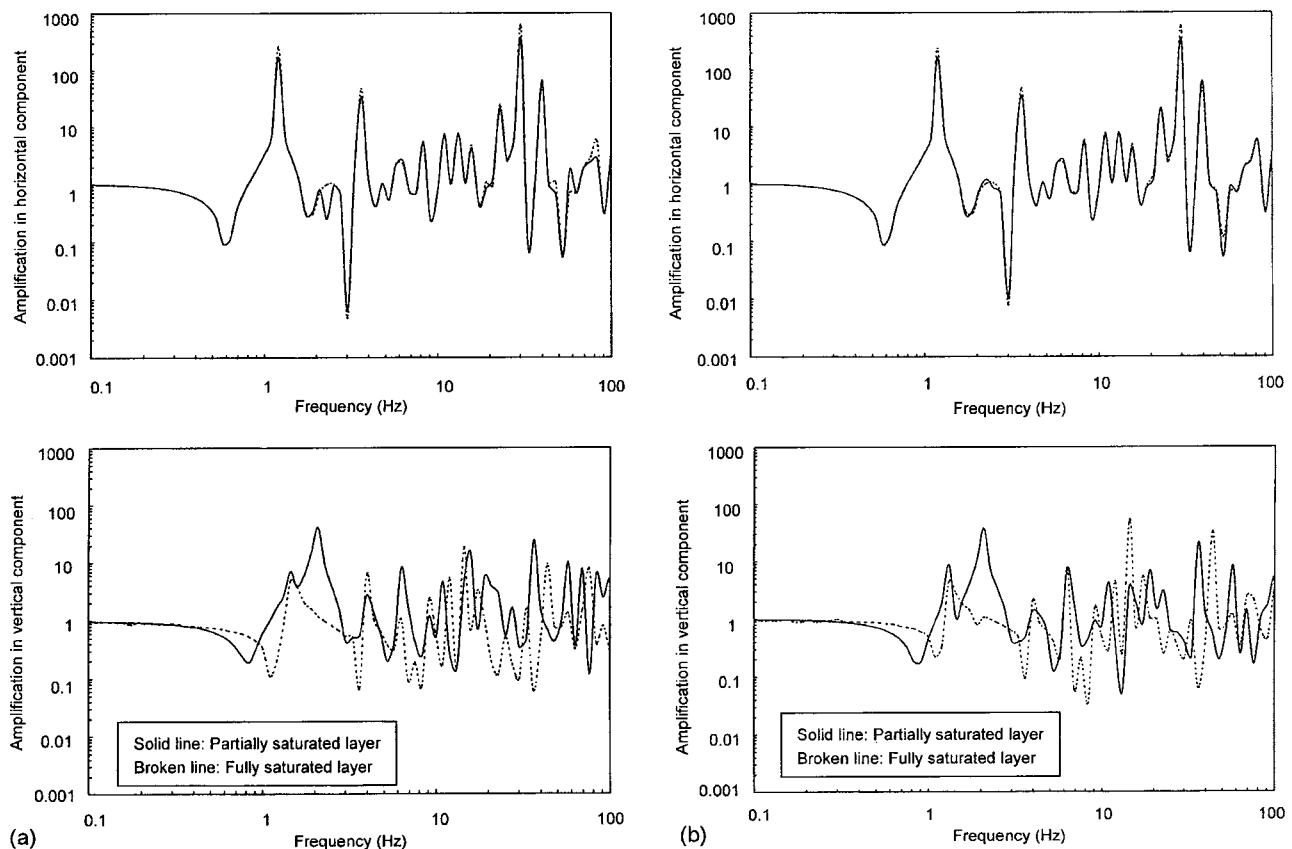
It is evident from Fig. 3 that, in the low-frequency range (i.e.,  $f/f_c \ll 1$ ), the velocities of the S wave and the first P wave are independent of frequency, while the velocity of the second P wave approaches zero. It is the second P wave that is responsible for the viscous damping effect of the saturated or partially

saturated sand and rock. Note that the frequencies of seismic waves are not high, certainly falling into this low-frequency range. Fig. 3 also indicates that the effect of saturation on the S wave velocity is negligible but very notable on the first P wave velocity; the latter may drop dramatically even for a slight decrease of full saturation.

### Effect of Saturation on Seismic Amplification

All four cases of saturation shown in Fig. 2 are investigated. Table 2 gives for each case the specific values of the degree of saturation used in the analysis. As mentioned before, in real situations, seismic waves are not vertically incident from the rock formation on the base of the surface layer, but generally with a small angle. For the first instance, here it is assumed that SV waves are incident at  $10^\circ$  to the vertical.

In Fig. 4(a), the soil amplification in both horizontal and vertical directions is shown for Cases 1 and 2. Referring to Table 2, in the two cases, the bedrock is assumed as fully saturated whereas the surface layer is either fully or partially saturated ( $S_r=100$  and  $95\%$ , respectively). The results for Cases 3 and 4, where the overlying sand layer is either fully or partially saturated but the bedrock is assumed as unsaturated, are shown in Fig. 4(b). Here, the amplification is conventionally defined as the ratios between the displacement amplitudes at the layer surface and at the layer bottom.



**Fig. 4.** Seismic amplification in horizontal and vertical directions: (a) Saturated bedrock and (b) unsaturated bedrock

**Table 3.** Velocities of P and S Waves in Sand and Bedrock

Sample	First P wave (m/s)		Second P wave (m/s)		S wave (m/s)	
	$S_r=100\%$	$S_r=95\%$	$S_r=100\%$	$S_r=95\%$	$S_r=100\%$	$S_r=95\%$
Sand	1562	255	0.38	0.07	145	145
Bedrock	1994	1136	0.05	0.003	655	655

Some interesting features exist in the four graphs in Fig. 4. First, the layered system displays a damping effect, although the constituents of both the sand and rock are assumed to be elastic. This damping effect is due to the viscous pore-fluid flow associated with the second P wave, as indicated by Fig. 3. Second, the saturation state of the bedrock has little influence on the amplification in either horizontal or vertical direction, especially at frequencies lower than 5 Hz. Third, no matter the bedrock is saturated or unsaturated, a change of the saturation state of the soil layer from full saturation to partial saturation may impose a marked impact on the amplification of vertical motion but a negligible influence on the horizontal amplification. For the horizontal amplification, the first peak always appears at the frequency of about 1.2 Hz. For the vertical motion, however, greater amplification will occur for an unsaturated soil layer, accompanied by a shift of peak frequency to the low-frequency end. One may note that this performance is reasonably consistent with the effect of saturation on the velocities of the (first) P and S waves shown in Fig. 3. For better comparison, Table 3 gives the values of the wave velocities in the sand and rock at the two different degrees of saturation.

The results shown in Fig. 4 also imply that the influence of the saturation state of the soil layer will disappear and thereby cannot be identified if the SV waves are conventionally assumed to propagate vertically. This is because under the condition of vertical incidence there is no SV-P conversion.

For all the four cases investigated, Fig. 5 presents the frequency-dependent amplitude ratios between vertical and horizontal components (denoted by  $V/H$ ) at the surface of the soil layer. The impact of the saturation state of the soil layer on the ( $V/H$ ) ratios can be clearly observed. In a typical range of seismic frequencies, a partially saturated layer can produce much higher values of the ( $V/H$ ) ratios than a fully saturated layer, implying the importance of in situ saturation conditions in interpreting the spectral ratios of surface ground motions or microtremors.

## Concluding Remarks

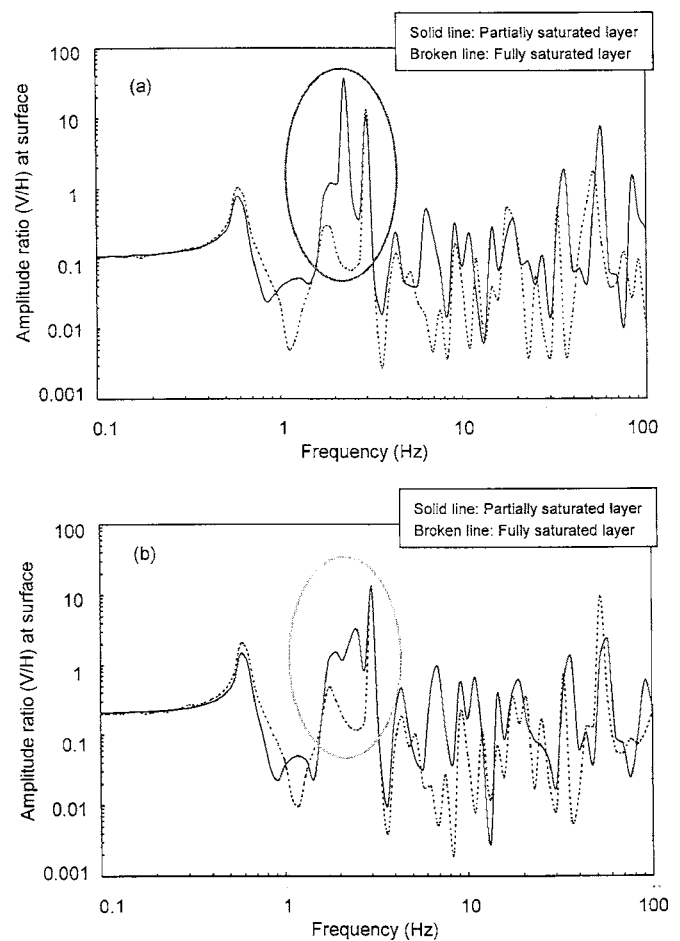
The frequency-dependent amplification of inclined SV waves in a surface soil layer overlying rock formation has been analyzed, and the influence of the saturation states of the surface soil and the bedrock on the amplification has been identified for both the horizontal and vertical components, as well as for the ratios between the two components. The results can be summarized as follows.

1. The saturation state of the bedrock has a minor influence on the amplification of the surface layer in either horizontal or vertical direction.
2. A change of the saturation state of the soil layer from full saturation to partial saturation may have a significant influence on vertical amplification but a negligible influence on horizontal amplification.
3. For typical seismic frequencies, an unsaturated soil layer may generate greater vertical amplification than a saturated

layer; it can also cause larger amplitude ratios between vertical and horizontal components at the surface.

4. Because of the presence of pore water, the layered system exhibits damping effect. The damping effect comes from the viscous flow of pore fluid associated with the propagation of the second P wave.
5. It is of importance to take into account the saturation condition of near-surface soils in interpreting ground motion data, especially vertical motion data.

Last, but not least, it should be mentioned that there is another important issue that is worth studying. The issue is concerning the effect of nonlinearity and pore water pressure on the ( $H/V$ ) spectral ratios. Yang et al. (2002) made an attempt to discuss this issue and reported some preliminary results. However, due to the complicated interrelations among the saturation state, pore pressure, and ground motion, research efforts over the years are needed in order to build deeper understanding of the problem.



**Fig. 5.** Frequency-dependent amplitude ratios ( $V/H$ ) at surface: (a) Saturated bedrock and (b) unsaturated bedrock



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