

Influence Zone for End Bearing of Piles in Sand

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Abstract: An attempt is made to establish an analytically based estimate of the influence zone surrounding the tip of a loaded pile in sand. In the framework of the cavity expansion theory and a confined local failure mechanism, explicit expressions are derived in which the sizes of the upward and downward influence zones are properly linked with the angle of shearing resistance, the stiffness, the volumetric strain, and the mean effective stress of the sand at the pile tip. Based on a series of parametric analyses, the mean range of the influence zone is suggested. For piles in clean sand, the influence zone above the pile tip is between 1.5 and $2.5D$ and the zone below the tip ranges from 3.5 to $5.5D$, where D is pile diameter. For piles in more compressible silty sand, the influence zone extends between 0.5 and $1.5D$ above the pile tip and between 1.5 and $3D$ below the tip. Because of its analytical nature, the present study may provide a meaningful insight into the current empirical interpretations.

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Introduction

The bearing capacity of piles in sand is identified as being the area of greatest uncertainty in foundation design (Randolph et al. 1994). For many years pile foundations have been designed, more or less successfully, using empirical approaches. Among these approaches, the cone penetration tests (CPT) based methods enjoy preference (Mayne et al. 1995; Randolph 2003). In particular, the empirical method developed by De Ruiter and Beringen, 1979, later known as the European/Dutch method, is now routinely used in design (Robertson et al. 1985; Briaud and Tucker 1988; Rollins et al. 1999). In this method, the ultimate base resistance of a pile is determined by the cone resistance over a zone of $0.7-4D$ below the pile tip and of $6-8D$ above the pile tip (Fig. 1), where D is the pile diameter.

The De Ruiter and Beringen approach is based on experience gained from the North Sea design practice. As indicated in their original paper, the influence zone was deduced from many load tests—CPT correlations and was to account for partial embedment of piles, i.e., pile tip being near the boundary between soft clay and underlying dense sand. This site-specific practice is now widely used for various site conditions; many of the sites do not bear the same characteristics as the North Sea conditions (e.g., Briaud and Tucker 1988; Rollins et al. 1999). When the De Ruiter and Beringen approach is applied to piles installed in sand where a strong soft over hard layering does not exist, a concern naturally arises over the rationality of the use of the influence zone of such

a form, simply because the practice implicitly assumes that pile end-bearing capacity is more dependent on the soil above the pile tip rather than the soil below the tip.

It may be argued that a fairly large influence zone above a pile tip is partly a reflection of the classical failure mechanism as that proposed by Meyerhof (1951, 1976) based on the bearing capacity theory for shallow foundations and small-scale model tests. However, model testing at a realistic ambient stress level indicates that the classical failure mechanism with slip planes propagating from below the foundation to the ground surface is not applicable to deep foundations (Miura 1985; White and Bolton 2005). The failure pattern at the tip of a pile in sand is more likely a type of local or punching failure rather than the general failure (Hirayama 1988; Randolph et al. 1994; Yasufuku and Hyde 1995; Yang et al. 2005).

In practical situations, the influence zone surrounding the tip of a loaded pile in sand is complicated and influenced by many factors, such as the angle of shearing resistance of the sand at pile

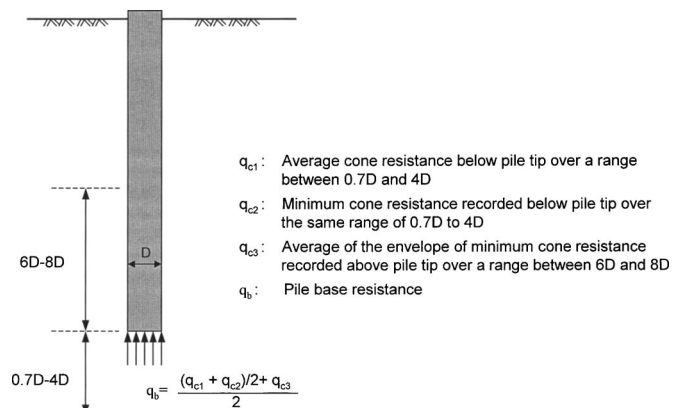


Fig. 1. Influence zone used in CPT-based estimation of pile end-bearing capacity

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tip, the stiffness of the sand, the relative density of the sand, the effective stress level at the pile tip, and the local heterogeneity (or layering). This study aims to develop an analytically based estimate of the influence zone by introducing a local failure mechanism and taking account of some of the key factors and thereby to offer an alternative insight into the current empirical interpretations.

Analysis

It has been well established that the deformation beneath the tip of a loaded pile resembles the expansion of a spherical cavity in an infinite medium (e.g., Vesic 1972; Randolph et al. 1994; Yasufuku and Hyde 1995). A comparison of various failure patterns (Yang et al. 2005) suggests that the confined local failure mechanism (Hirayama 1988; Yasufuku and Hyde 1995), as shown in Fig. 2, can provide a fairly good prediction for the end-bearing capacity of displacement piles in sand. This failure pattern is thus adopted for developing an estimate of the influence zone around the pile tip.

Referring to Fig. 2, it is assumed that the limit pressure for cavity expansion, p_u , acts on the spherical surface AEC and that the active earth pressure, σ_A , acts on the surface AF . ACF forms part of the wedge under the pile, and the angle ψ is determined by

$$\psi = 45 + \frac{\phi'}{2} \quad (1)$$

where ϕ' = the effective friction angle of the sand beneath the pile tip.

From Fig. 2 a relationship between the cavity expansion pressure, p_u , and the end-bearing capacity of the pile, q_b , can be established as follows (Yasufuku and Hyde 1995):

$$q_b = \left(\frac{1}{1 - \sin \phi'} \right) p_u \quad (2)$$

Note that in the theory of cavity expansion q_b is a real ultimate capacity that differs from the nominal ultimate capacity defined at a specific settlement criterion in pile load tests (e.g., 10% of pile diameter). Care should therefore be used in comparing the capacity determined from a load test with that determined from a cavity expansion method.

At the limit state the cavity has a radius R_u and the plastic zone around the cavity extends to a radius R_p . Beyond this radius, the

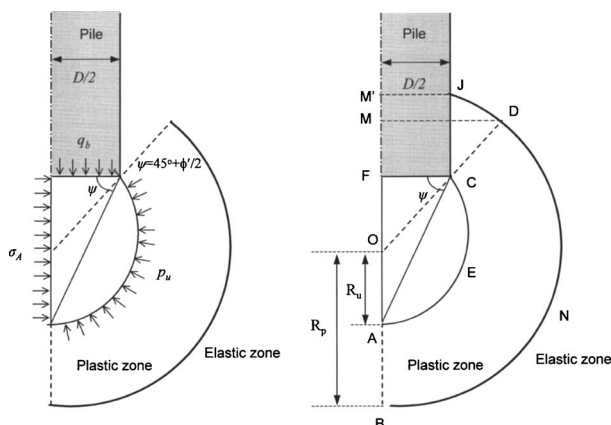


Fig. 2. Influence zone assumed for piles in sand

rest of the soil mass remains in a state of elastic equilibrium. Thus it is appropriate to define that the zone of influence is within the boundary ($BNDJ$) between the plastic and elastic zones. From Fig. 2 one may readily arrive at

$$R_u = \frac{D}{2 \cos \phi'} \quad (3)$$

Letting $\zeta = R_p/R_u$, the influence zone below the pile tip can be given by

$$I_{FD} = FB = \frac{D}{2} \left(\tan \phi' + \frac{\zeta}{\cos \phi'} \right) \quad (4)$$

and the influence zone above the pile tip is determined by

$$I_{FU} = FM' \approx FM = \frac{D}{2} (\zeta - 1) \tan \phi' \quad (5)$$

where the approximation is introduced for mathematical convenience in derivation.

The relative radius of the plastic zone, ζ , can be derived analytically in the framework of cavity expansion theory (Vesic 1972) as

$$\zeta^3 = \frac{1 + \Delta}{\eta I_r + \Delta} \quad (6)$$

in which

$$\eta = \frac{3 \cos \phi'}{3 - \sin \phi'} \quad (7)$$

$$I_r = \frac{G}{p'_0 \tan \phi'} \quad (8)$$

Here Δ = average volumetric strain in the plastic zone; I_r = rigidity index; G = shear modulus of the soil; and p'_0 = initial mean effective stress at the level of pile tip. Noting that for most cases where $\Delta < 0.1$ and $0 < \phi' < 45^\circ$, $\eta \approx 1$, and $\sqrt[3]{1 + \Delta} \approx 1$, the expression in Eq. (6) can further be simplified as

$$\zeta = \frac{R_p}{R_u} = \sqrt[3]{\frac{I_r}{1 + I_r \Delta}} \quad (9)$$

The relationship expressed above is graphically shown in Fig. 3. Clearly, the relative radius of plastic zone increases with increasing the rigidity index; at a given rigidity index, the relative radius decreases with the average volumetric strain in the plastic zone. The significant influence of the average volumetric strain implies that one should not simply assume $\Delta = 0$ in estimating the influence zone.

With Eqs. (8) and (9) at hand and by a straightforward manipulation, Eqs. (4) and (5) can be given in the following normalized form, i.e., the influence zone below the pile tip

$$\left(\frac{I_{FD}}{D} \right) = \frac{1}{2} \left(\tan \phi' + \sqrt[3]{\frac{G}{G\Delta + p'_0 \tan \phi' \cos \phi'}} \right) \quad (10)$$

the influence zone above the pile tip

$$\left(\frac{I_{FU}}{D} \right) = \frac{1}{2} \left(\sqrt[3]{\frac{G}{G\Delta + p'_0 \tan \phi'}} - 1 \right) \tan \phi' \quad (11)$$

In the above two expressions the sizes of the upward and downward influence zones are properly linked with the effective friction angle, shear modulus, volumetric strain, and mean effective stress of the soil at the pile tip.

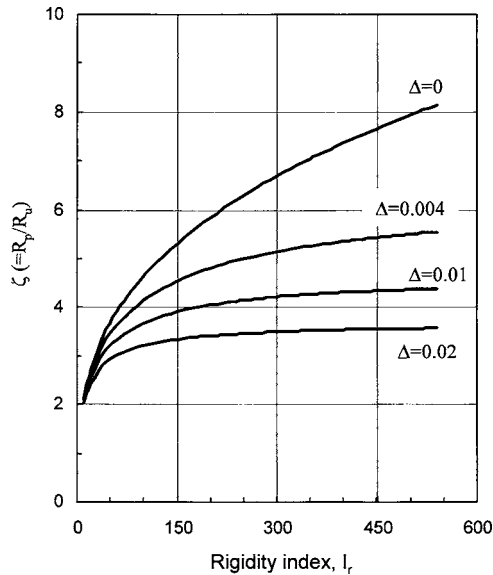


Fig. 3. Relative radius of plastic zone as function of rigidity index

Compressibility of Sand

In engineering practice, the shear modulus of sand and in situ mean effective stress may vary more significantly than the angle of shearing resistance. For instance, a clean silica sand may have its shear modulus much greater than a crushable silty sand, and the level of the mean stress at the tip of a very long pile can be several times larger than the stress at the tip of a short pile. These variations may have a marked impact on the compressibility of sand and, subsequently, on the zone of influence surrounding the pile tip. Moreover, the variations of these quantities are responsible in part for the scale effect in physical modeling. Therefore, it is important to take account of this influence in estimating the range of influence zone.

The mean effective stress and void ratio (or relative density) are recognized as two most influencing factors for sand behavior (e.g., Yang and Li 2004). A number of empirical correlations have been proposed to relate the shear modulus with these two factors (see Ishihara 1996). Considering that in many practical cases sands may have less or more fines contents, the correlation proposed by Lo Presti (1987) is employed here

$$\frac{G_0}{p_a} = m \exp(\chi D_r) \left(\frac{\sigma'_0}{p_a} \right)^n \quad (12)$$

in which the parameter m reflects the effect of fines content; σ'_0 =effective isotropic confining pressure; p_a =reference pressure (100 kPa); D_r =relative density of sand; and χ and n =two parameters with typical values of 0.7 and 0.5, respectively.

Considering the suggestion of Randolph (1994) and to a first approximation, the following relations are used to approximately characterize the shear modulus for two representative types of sand:

Clean sand having a fines content less than 5%

$$\frac{G_0}{p_a} = 400 \exp(0.7D_r) \left(\frac{\sigma'_0}{p_a} \right)^{0.5} \quad (13)$$

Silty sand having a fines content of 15–30%

$$\frac{G_0}{p_a} = 75 \exp(0.7D_r) \left(\frac{\sigma'_0}{p_a} \right)^{0.5} \quad (14)$$

Using Eqs. (13) and (14), the variations of the shear modulus G_0 with the relative density and confining pressure are shown in Fig. 4 for clean sand and silty sand. As can be seen, for either clean or silty sand, the shear modulus increases with increasing the relative density and confining pressure. At a given density and confining pressure, the compressibility of silty sand is significantly greater than that of clean sand.

It should be noted that G_0 is an initial tangent shear modulus determined from small-strain testing where the shear strain is on the order of 10^{-5} – 10^{-6} . For the problem concerned, the strain level is certainly much higher than this value and the stress–strain response is nonlinear. A large number of experimental studies have examined the strain-level dependency of shear modulus. Here, the empirical correlations proposed by Ishibashi and Zhang (1993) are used

$$\frac{G}{G_0} = \alpha (\sigma'_0)^\beta \quad (15)$$

$$\alpha = \frac{1}{2} + \frac{1}{2} \tanh \left[\ln \left(\frac{0.000102 + n}{\gamma} \right)^{0.492} \right] \quad (16)$$

$$\beta = 0.272 \left\{ 1 - \tanh \left[\ln \left(\frac{0.000556}{\gamma} \right)^{0.4} \right] \right\} \exp(-0.0145PI^{1.3}) \quad (17)$$

$$n = \begin{cases} 0 & \text{for } PI = 0 \\ 3.37 \times 10^{-6} PI^{1.404} & \text{for } 0 < PI \leq 15 \\ 7.0 \times 10^{-7} PI^{1.976} & \text{for } 15 < PI \leq 70 \end{cases} \quad (18)$$

where G =shear modulus at a shear strain γ ; and PI =plasticity index of soil. As it includes the quantity PI , the above correlations provide convenience in estimating the strain-dependent shear modulus of sands of different fines contents.

Using Eqs. (15)–(18), Fig. 5(a) shows the shear modulus reduction curves for clean sand under different values of confining

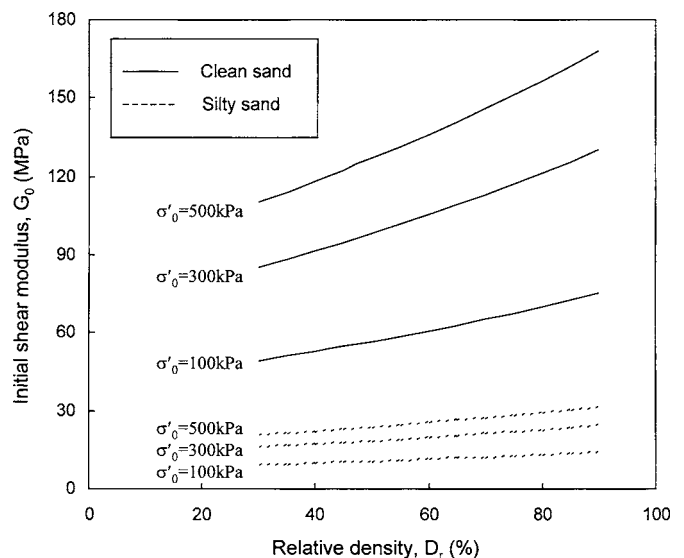


Fig. 4. Initial shear modulus of sand as function of relative density and confining pressure

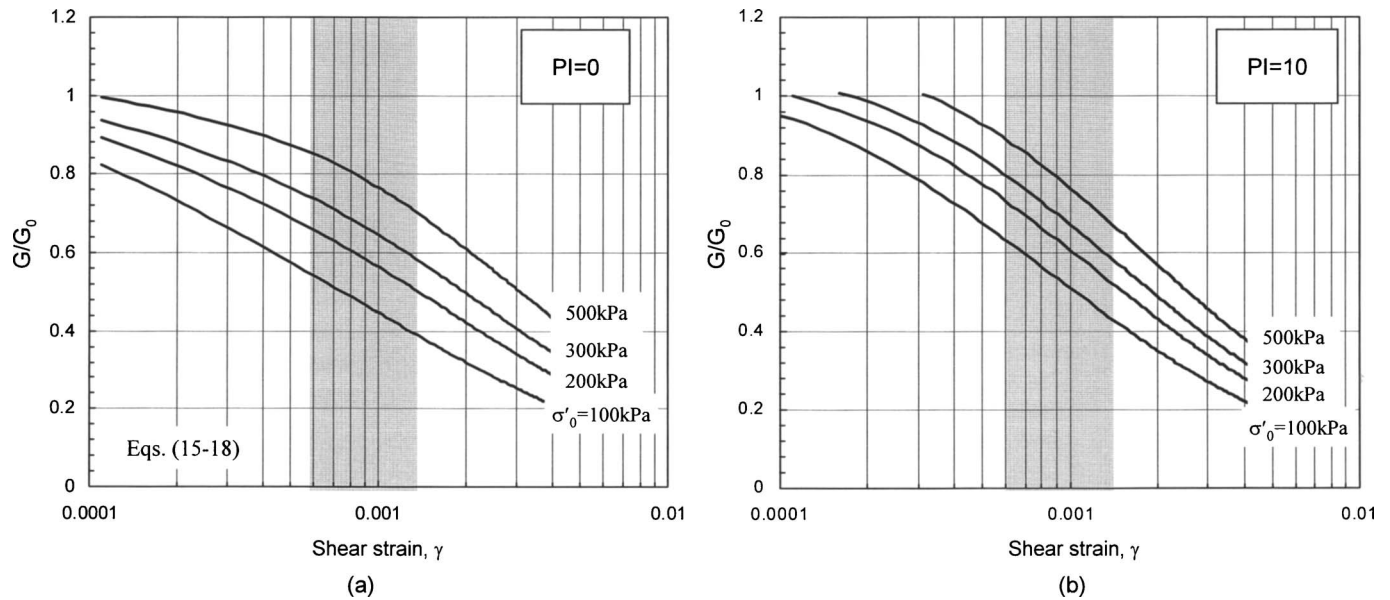


Fig. 5. Degradation of shear modulus: (a) clean sand; (b) silty sand

pressure. The plasticity index of clean sand is taken as zero. For silty sand with fines content over 15%, it is assumed that the sand has low plasticity (PI=10), and the calculated shear modulus reduction curves are presented in Fig. 5(b). Considering that the typical range of shear strains in elasto-plastic deformations is between 10^{-4} and 10^{-2} (Ishihara 1996), it is assumed, to a first

approximation, that the average strain level involved in the present analysis is in the order of 0.1%. At this strain level the shear modulus of either clean sand or silty sand under a confining pressure of 100 kPa is approximately 45% of that at a very small strain (Fig. 5).

The average volumetric strain, Δ , also plays a role in estimat-

Table 1. Relative Radius of Plastic Zone in Clean Sand

ϕ' (degrees)	$D_r=75\%$			$D_r=55\%$			$D_r=35\%$		
	I_r	Δ	ζ	I_r	Δ	ζ	I_r	Δ	ζ
$p'_0=100$ kPa									
25	652.54	0.000429	7.99	567.29	0.000552	7.56	493.18	0.000711	7.15
27	597.19	0.000503	7.71	519.17	0.000648	7.30	451.34	0.000833	6.90
29	548.94	0.000586	7.46	477.23	0.000754	7.05	414.88	0.000970	6.66
31	506.41	0.000677	7.22	440.25	0.000872	6.83	382.74	0.001121	6.44
33	468.56	0.000779	7.00	407.34	0.001002	6.61	354.13	0.001290	6.24
35	434.56	0.000892	6.79	377.79	0.001148	6.41	328.43	0.001477	6.05
37	403.80	0.001018	6.59	351.04	0.00131	6.22	305.18	0.001686	5.86
39	375.76	0.001159	6.40	326.67	0.001491	6.03	283.99	0.001919	5.68
41	350.04	0.001317	6.21	304.31	0.001694	5.86	264.55	0.002180	5.52
43	326.30	0.001494	6.03	283.67	0.001923	5.68	246.61	0.002474	5.35
45	304.28	0.001695	5.86	264.53	0.002180	5.52	229.97	0.002805	5.19
$p'_0=500$ kPa									
25	499.34	0.000695	7.18	434.11	0.000894	6.79	377.39	0.001150	6.41
27	456.99	0.000815	6.93	397.29	0.001049	6.54	345.38	0.001349	6.18
29	420.07	0.000948	6.70	365.19	0.001220	6.32	317.48	0.001570	5.96
31	387.52	0.001097	6.48	336.90	0.001411	6.11	292.88	0.001815	5.76
33	358.55	0.001261	6.27	311.71	0.001623	5.92	270.99	0.002088	5.57
35	332.54	0.001444	6.08	289.10	0.001858	5.73	251.33	0.002391	5.39
37	309.00	0.001648	5.89	268.63	0.002121	5.55	233.54	0.002729	5.22
39	287.54	0.001876	5.72	249.98	0.002414	5.38	217.32	0.003106	5.06
41	267.86	0.002132	5.54	232.87	0.002743	5.22	202.44	0.003529	4.91
43	249.70	0.002419	5.38	217.08	0.003112	5.06	188.72	0.004004	4.75
45	232.85	0.002743	5.22	202.43	0.003529	4.90	175.98	0.004541	4.61

Note: p'_0 =initial mean effective stress; D_r =relative density; I_r =rigidity index; Δ = average volumetric strain; and $\zeta=R_p/R_u$.

Table 2. Relative Radius of Plastic Zone in Silty Sand

ϕ' (degrees)	$D_r=75\%$			$D_r=55\%$			$D_r=35\%$		
	I_r	Δ	ζ	I_r	Δ	ζ	I_r	Δ	ζ
$p'_0=100$ kPa									
25	122.35	0.008735	3.90	106.37	0.011239	3.64	92.47	0.01446	3.41
27	111.97	0.010246	3.74	97.34	0.013183	3.49	84.63	0.016961	3.26
29	102.93	0.011924	3.59	89.48	0.015341	3.35	77.79	0.019738	3.13
31	94.95	0.013787	3.45	82.55	0.017738	3.22	71.76	0.022822	3.01
33	87.85	0.015856	3.32	76.38	0.020401	3.10	66.40	0.026247	2.89
35	81.48	0.018158	3.20	70.84	0.023362	2.99	61.58	0.030058	2.78
37	75.71	0.020724	3.09	65.82	0.026663	2.88	57.22	0.034305	2.68
39	70.46	0.023590	2.98	61.25	0.030351	2.78	53.25	0.039050	2.58
41	65.63	0.026802	2.89	57.06	0.034483	2.68	49.60	0.044366	2.50
43	61.18	0.030412	2.78	53.19	0.039128	2.58	46.24	0.050342	2.40
45	57.05	0.034488	2.68	49.60	0.044372	2.49	43.12	0.057089	2.32
$p'_0=500$ kPa									
25	93.63	0.014140	3.43	81.40	0.018193	3.20	70.76	0.023406	2.99
27	85.68	0.016586	3.28	74.49	0.021339	3.06	64.76	0.027455	2.86
29	78.76	0.019302	3.15	68.47	0.024833	2.94	59.53	0.031951	2.74
31	72.66	0.022317	3.03	63.17	0.028713	2.82	54.92	0.036942	2.63
33	67.23	0.025667	2.91	58.45	0.033023	2.71	50.81	0.042487	2.52
35	62.35	0.029393	2.80	54.21	0.037817	2.61	47.12	0.048656	2.43
37	57.94	0.033546	2.70	50.37	0.043161	2.51	43.79	0.055530	2.34
39	53.91	0.038186	2.60	46.87	0.04913	2.42	40.75	0.063211	2.25
41	50.22	0.043384	2.51	43.66	0.055818	2.33	37.96	0.071815	2.17
43	46.82	0.049229	2.42	40.70	0.063338	2.25	35.38	0.081490	2.09
45	43.66	0.055826	2.33	37.96	0.071826	2.17	33.00	0.092411	2.01

Note: p'_0 =initial mean effective stress; D_r =relative density; I_r =rigidity index; Δ =average volumetric strain; and $\zeta=R_p/R_u$.

ing the radius of the plastic zone. Generally, it can be determined based on the isotropic and triaxial compression tests that describe the compressibility of sand due to compression and distortion (Vesic 1972). To avoid the difficulty of experimentally determining the volumetric strain, Yasufuku et al. (2001) suggest an empirical relation for sands varying from stiff siliceous sand to compressible carbonate sand

$$\Delta = 50(I_r)^{-1.8} \tag{19}$$

in which I_r =rigidity index defined earlier. The above relation indicates that the average volumetric strain drops dramatically with increasing the rigidity index.

Now, one is able to calculate the rigidity index, the average volumetric strain and the relative radius of the plastic zone for

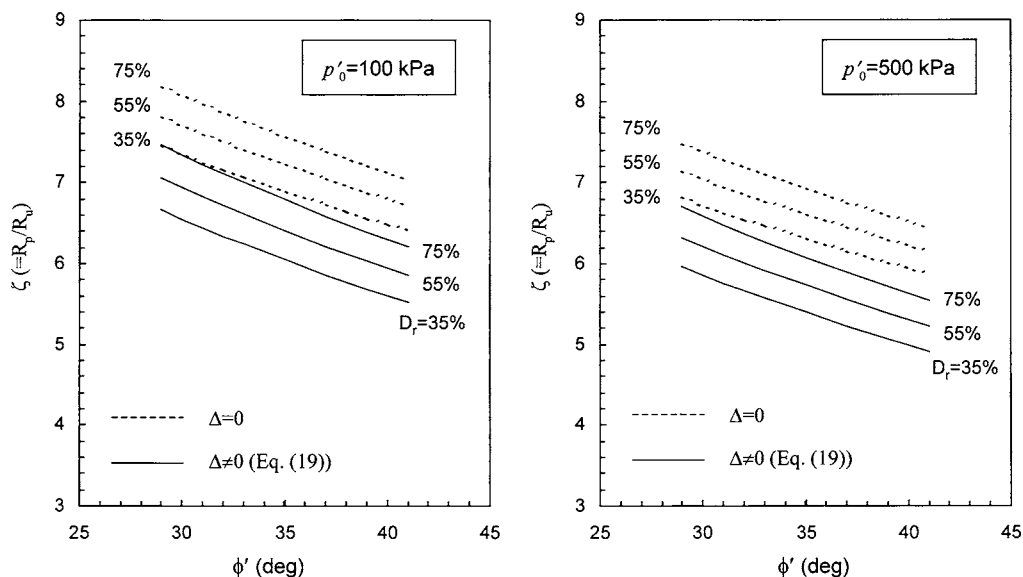


Fig. 6. Influence of compressibility on relative radius of plastic zone in clean sand

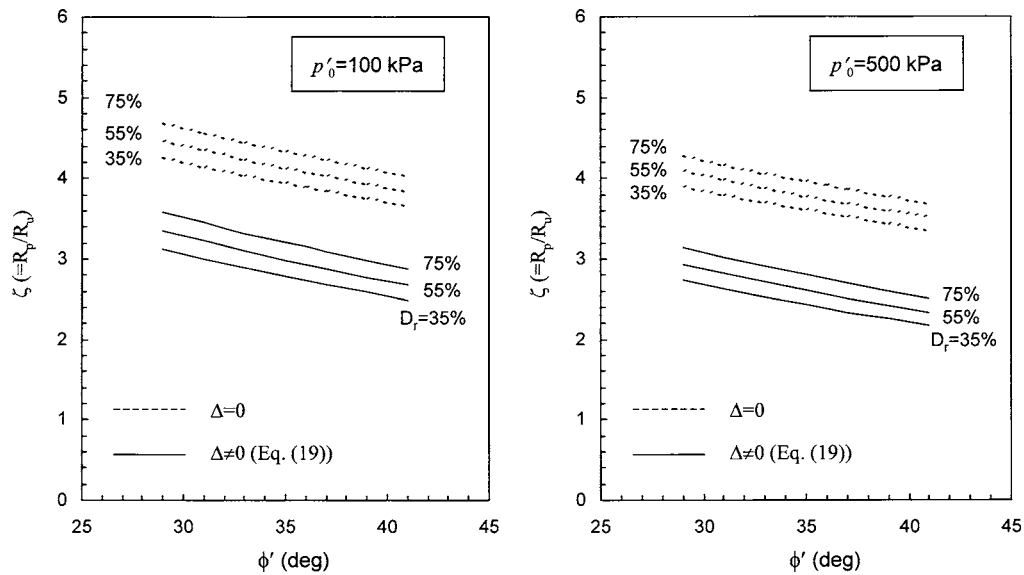


Fig. 7. Influence of compressibility on relative radius of plastic zone in silty sand

different values of mean effective stress, relative density and friction angle. Tables 1 and 2 give the calculated results for the defined clean sand and silty sand, respectively. For both types of sand, two cases of the mean effective stress level, i.e., 100 and 500 kPa, are considered. These values are used to roughly represent the stress levels for short piles ($L \sim 10$ m) and long piles ($L \sim 50$ m, where L is the embedded length of a pile), respectively. Comparison of the two tables indicates that the silty sand is more compressible than the clean sand, with the value of Δ being one to two orders larger than that of clean sand. Under otherwise

identical conditions, the relative radius of the plastic zone for silty sand is less than one half of that for clean sand. The typical values of the ratio R_p/R_u are approximately 5–8 for clean sand and 2–4 for silty sand.

For a specific case ($p'_0 = 100$ kPa and $\Delta = 0$) and without numerical justification, Vesic (1972) gives a rough estimate that the typical value of R_p/R_u for sand is between 4 and 6. By comparison, this estimate falls within the range derived in this study and is closer to the values for clean sand. A point that deserves noting

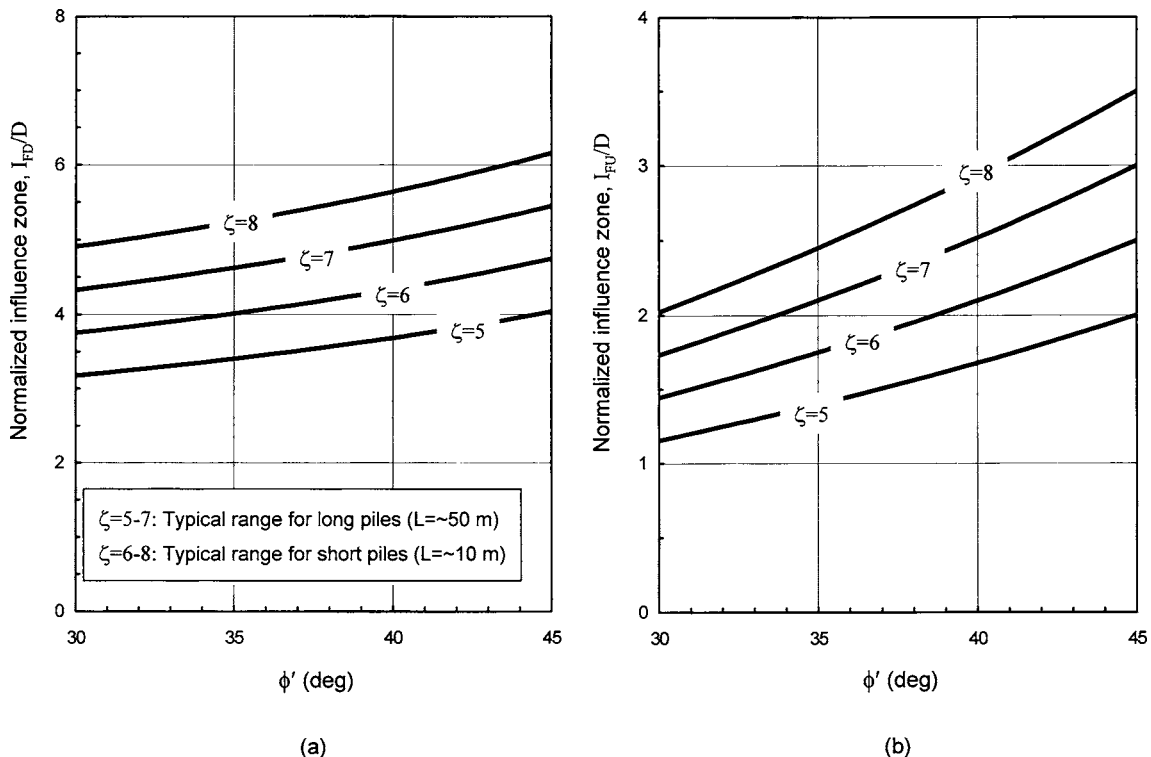


Fig. 8. Influence zone for piles in clean sand: (a) below pile tip; (b) above pile tip

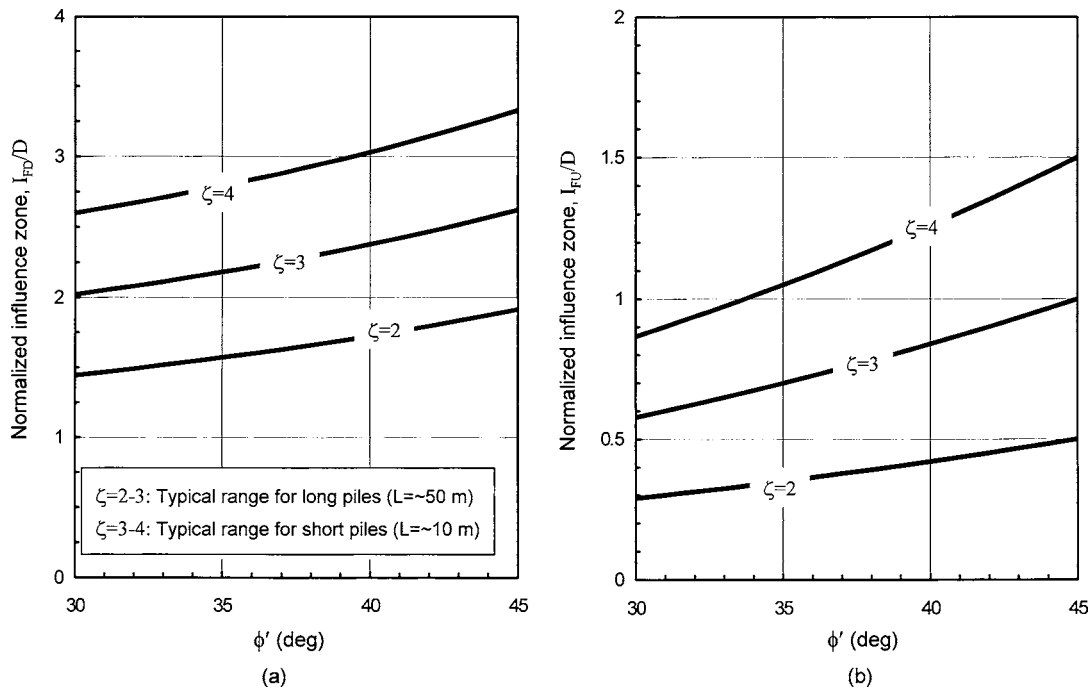


Fig. 9. Influence zone for piles in silty sand: (a) below pile tip; (b) above pile tip

herein is that a significant increase in the value of R_p/R_u may arise from the neglect of the volumetric strain Δ , as shown in Figs. 6 and 7.

Range of Influence Zone

Using the derived expressions [Eqs. (10) and (11)], the sizes of the influence zones below and above the pile tip are calculated, and shown against the friction angle for the case of clean sand in Fig. 8. The results for silty sand are plotted in Fig. 9. It is noted

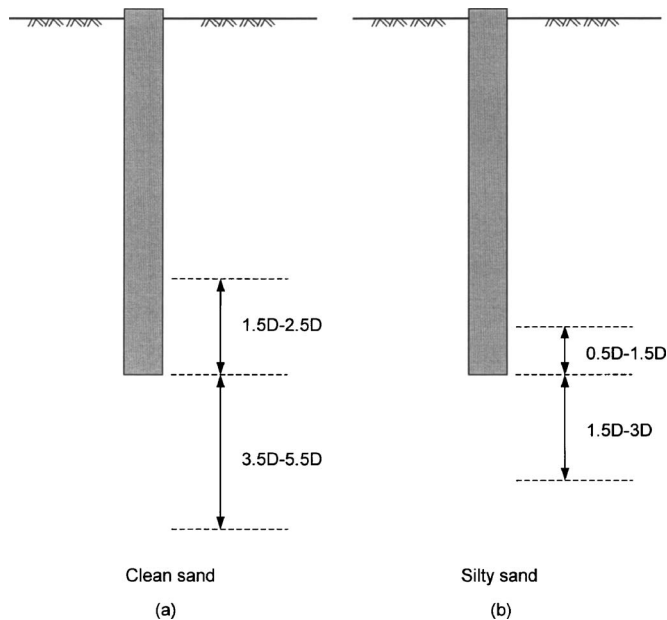


Fig. 10. Mean range of influence zone for pile in sand: (a) piles in clean sand; (b) piles in silty sand

that, depending on pile length and soil properties, the influence zone below pile tip can vary from 1.5 to as high as $6D$ while the zone above pile tip can range from less than 0.5 to as large as $3.5D$.

There are some observations that are of particular interest:

1. The compressibility of sand has a marked impact on the size of influence zone;
2. The influence zone below the pile tip is larger than the zone above pile tip;
3. The influence zone surrounding the tip of a long pile is generally smaller than that surrounding the tip of a short pile;
4. An increase in the angle of shearing resistance causes an increase in the sizes of both the upper and lower zones; and
5. The angle of shearing resistance has a more significant impact on the upper zone than the lower zone.

Based on the results for a wide range of in situ mean stresses, relative densities, and friction angles, and taking into considerations the uncertainties that may be introduced in the analysis, Fig. 10 suggests the mean range of the influence zone surrounding the pile tip. For piles in clean sand, the influence zone above the pile tip is between $1.5D$ and $2.5D$ and the zone below the tip ranges from $3.5D$ to $5.5D$. In comparison, the zone of influence for piles in more compressible silty sand extends between $0.5D$ and $1.5D$ above the pile tip and between $1.5D$ and $3D$ below the tip.

Discussion

There is another widely accepted CPT-based empirical method for pile design, known as the LCPC method. This method was developed by Laboratoire Central des Ponts et Chaussées in France (Bustamante and Gianceselli 1982). In this method, the end-bearing capacity of a driven pile is determined by the cone resistance in a range of $1.5D$ below the pile tip to $1.5D$ above the pile tip. It is interesting to note, by comparison, that the estimate derived from this study covers a *reasonably balanced* range be-

tween those used in the LCPC method and the European method.

It is out of the scope of this study to have a detailed evaluation of the European/Dutch method and the LCPC method for pile capacity prediction. But in line with the conception of the present analysis, it seems to be more rational to assume the influence zone above the pile tip to be $1.5D$ than to be $6-8D$ for piles at predominantly sand sites, and the size of $1.5D$ appears to be closer to our theoretical estimate. In this regard, observations on some of the recent case studies (Briaud and Tucker 1988; Rollins et al. 1999; Puppala and Moalim 2002), which were aimed at evaluating various CPT-based pile design methods by using field load tests, are worth mentioning. The results of these studies consistently indicate that the LCPC method can generally provide better predictions of pile capacity than the European/Dutch method, especially for piles in sandy soil.

The effect of local heterogeneity (or layering) is an interesting issue that is not examined in this study. This effect is expected to be particularly profound in cases where pile tip is close to a boundary between an overlying weak layer and the underlying stiff bearing stratum. Detailed discussion on this issue is out of the scope of the present study. Reference can be made to Meyerhof (1976) and Hryciw and Shin (2004) for more information.

Conclusions

An attempt has been made to establish an analytically based estimate of the influence zone surrounding the tip of a loaded pile in sand. Explicit expressions have been established in which the sizes of both upward and downward zones are properly linked with the friction angle, stiffness, volumetric strain, and in situ mean effective stress of the sand at the pile tip. Based on the numerical results for a wide range of friction angles, shear moduli, and mean effective stresses, the mean range of the influence zone around the pile tip is suggested. For piles in clean sand with less than 5% fines, the influence zone above the tip is between 1.5 and $2.5D$ and the zone below the tip extends from 3.5 to $5.5D$. For piles in more compressible silty sand with fines content over 15%, the upward zone is between 0.5 and $1.5D$ and the downward zone ranges from 1.5 to $3D$.

Finally, it should be mentioned that the influence zone is explained in this study as the extent of the plastic zone in the cavity expansion theory where the soil is assumed to be homogeneous. While providing an alternative insight into the current empirical interpretations, the present study should not be considered as design guidance. Several interesting issues such as the effects of heterogeneity and the strain-dependent stiffness remain to be examined in detail in future studies.

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Notation

The following symbols are used in this technical note:

- D = pile diameter;
- D_r = relative density of sand;
- G = shear modulus;

- G_0 = small-strain shear modulus;
- I_{FD} = downward influence zone;
- I_{FU} = upward influence zone;
- I_r = rigidity index;
- L = pile embedded length;
- PI = plasticity index of soil;
- p_a = reference pressure (=100 kPa);
- p_u = limit cavity expansion pressure;
- p'_0 = in situ mean effective stress;
- q_b = ultimate unit end resistance of pile;
- R_i = initial radius of cavity;
- R_p = radius of plastic zone;
- R_u = radius of cavity;
- γ = shear strain;
- Δ = average volumetric strain;
- ζ = relative radius of plastic zone ($=R_p/R_u$);
- σ'_0 = effective confining pressure; and
- ϕ' = effective friction angle.

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