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A simple approach to integration of acceleration data for dynamic soil–structure interaction analysis

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Abstract

Accumulating experience indicates that direct integration of the ground acceleration data provided for seismic soil–structure interaction analysis often causes unrealistic drifts in the derived displacement. The drifts may have a significant effect on large-scale interaction analysis in which the displacement excitation is required as an input. This paper proposes a simple approach to integration of the acceleration to acquire a realistic displacement–time series. In this approach, the acceleration data is firstly baseline-corrected in the time domain using the least-square curve fitting technique, and then processed in the frequency domain using a windowed filter to further remove the components that cause long-period oscillations in the derived displacement. The feasibility of the proposed approach is assessed using several examples and comparisons are made between the results obtained using the proposed scheme and those using other complicated procedures.

Keywords: Baseline correction; Dynamic response analysis; Signal processing; Soil-structure interaction; Strong ground motion

1. Introduction

It is widely recognized that in many cases the dynamic interaction between a structure and the supporting soil has a profound effect on the response of the structure to earthquake loading and cannot be simply neglected. There are currently two major methods for analyzing seismic soil–structure interaction [1]: the direct method and the substructure method. While the direct method is a conceptually easier way to model the entire soil–structure system in a single step, the substructure method is computationally more efficient [2]. In the substructure method, it is required to estimate the forces acting on the soil–structure interface based on the dynamic-stiffness coefficients that represent the significant features of the unbounded soil. In other words, the earthquake excitations in terms of velocity and displacement need to be evaluated to calculate the forces.

A direct approach to acquiring the velocity and displacement excitations from the earthquake acceleration is to make use of the inherent relations between the displacement, velocity and acceleration by assuming zero initial conditions:

$$\dot{u}_g(t) = \int_0^t \ddot{u}_g(\tau)\,d\tau, \tag{1}$$

$$u_g(t) = \int_0^t \dot{u}_g(\tau)\,d\tau, \tag{2}$$

where $\ddot{u}_g(t)$ is the ground acceleration time series that is either recorded or synthesized, $\dot{u}_g(t)$ and $u_g(t)$ are the velocity and displacement time series, respectively. Eqs. (1) and (2) can be conveniently expressed in the discrete form as

$$\dot{u}_g(t) = \sum_{i=1}^{N} \frac{1}{2}(\ddot{u}_g(i-1) + \ddot{u}_g(i))\,\Delta\tau, \tag{3}$$

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$$u_g(t) = \sum_{i=1}^{N} \frac{1}{2}(\dot{u}_g(i-1) + \dot{u}_g(i))\,\Delta\tau, \tag{4}$$

where N is the number of sampling points in the acceleration time series, $\Delta\tau$ is the integral time step and is required to be sufficiently short to satisfy the condition of convergence.

Accumulating experience indicates, however, that the direct integration of the acceleration data often causes unrealistic drifts in the velocity and displacement, as can be seen in Fig. 1 where an acceleration record obtained during the 1994 Northridge Earthquake is given as an example. Use of the drifted velocity and displacement time series as input motions may have a profound effect on large-scale soil–structure interaction analysis, being manifested by computational instability or inaccurate numerical results. This may occur particularly in the case where the structure concerned has a long span and the traveling wave effect becomes significant [3].

The reason for the drifts in velocities and displacements derived by integrating acceleration records has been of concern in engineering seismology for a long time [4–10], and many potential sources have been identified. For example, it is generally agreed that the mechanical or electrical hysteresis in the sensor can cause an offset occurring in the acceleration records. Even a small offset in accelerations can produce significant drifts in velocities and displacements. Another major source of unrealistic drifts in velocities and displacements may come from the accumulation of the random noise in accelerations resulting from the single and double integration.

Various correction schemes have been proposed to deal with the problems of baseline offsets. In view of the difficulty of understanding the exact nature of the baseline shifts, there is no universal correction scheme at present that can be blindly applied [9]. One of the schemes commonly used in processing the accelerograms is to assume the zero-acceleration baseline to be of the polynomial form, the constants of which are determined by minimizing the mean square computed velocity [4]. A modification to this simple procedure was suggested by Boyce [11] to take into account the starting delay on the accelerograms. Noting that the baseline correction was not adequate in some cases, particularly for long acceleration records, Trifunac [5] proposed a processing scheme involving multiple baseline corrections and high-pass filtering of the acceleration and velocity time series, which was shown to be largely independent of the record length. The scheme has been developed as a standard procedure of strong motion data processing [10].

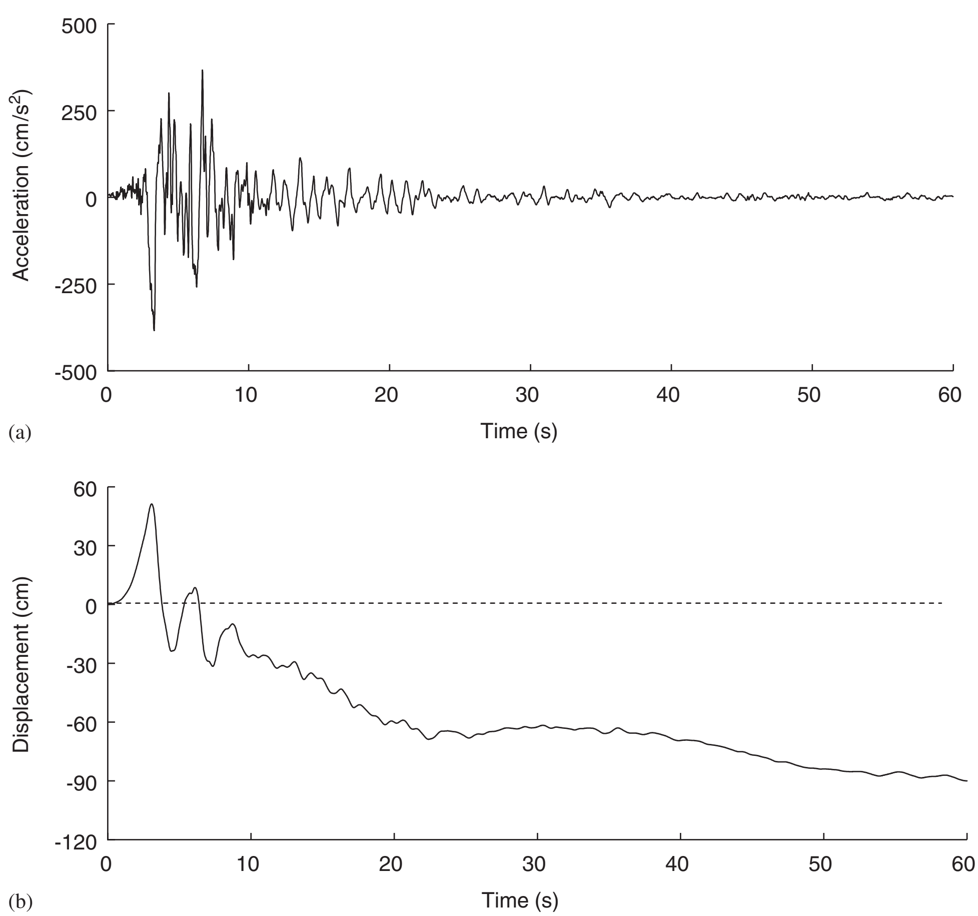

Fig. 1. Acceleration record obtained during the 1994 Northridge, California, earthquake and the displacement time series derived from double integration of the acceleration: (a) acceleration record, (b) integrated displacement.

Trujillo and Carter [12] suggested an approach in which the problem of baseline offsets was expressed as a minimization problem. That is, given a set of measured accelerometer data a_i^* $(i = 1,\ N)$, one wishes to find another set, a_i, that is close to a_i^* but on integration produces a small terminal velocity. The correction is made by using a dynamic programming formulation in which a set of weighting factors are involved. However, the effect of the weighting factors on the correction is unclear.

Based on the notion that the baseline of the acceleration record had random shifts during the interval of strong shaking and the shifts could be represented by an average baseline correction over this interval, Iwan et al. [7] suggested a correction scheme in which the constraint that the average velocity at the end of the record be zero was employed. The scheme involved fitting a straight line to velocity for some portion after the strong shaking ceased and then connecting this straight line with another line starting from zero velocity at time t_1 and joining the fitted line at the later time t_2. The correction of the acceleration was then made by subtracting the slopes of the velocity lines.

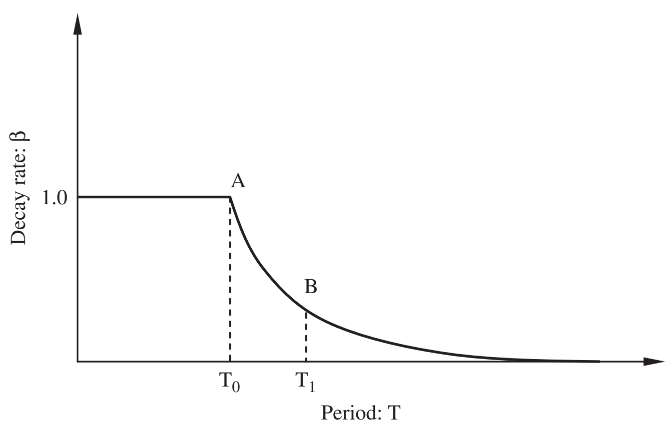

Fig. 2. Windowed filter used in this study.

It has been found that the degree of drifts in the displacements resulting from double integration of the corrected acceleration depends strongly on the processing parameter t_2 [9]. Even care is used in the selection of the processing parameter t_2, the baseline correction using the scheme may still yield unreasonable displacements, as indicated by laboratory shaking table tests on digital instruments [13]. While an improvement can be made by applying a high-pass filter to the corrected acceleration records, undesirable long-period oscillations still occur in the displacements derived from the filtered acceleration data in some cases.

Based on the laboratory observations, Zhou et al. [13] suggested a correction procedure that also involved multiple baseline corrections and filtering. The first step was to make a baseline correction to the acceleration record by using the scheme of Iwan et al. [7]. Second, a baseline correction was made to the velocity integrated from the baseline-corrected acceleration. Third, the displacement was derived by integrating the corrected velocity. And finally, a high-pass filter was applied to the displacement to further remove the long-period oscillations. While the scheme was shown to yield reasonable-looking displacements for all types of digital instruments tested, one should note that the inherent relations between the acceleration, velocity and displacement as shown in Eqs. (1) and (2) are

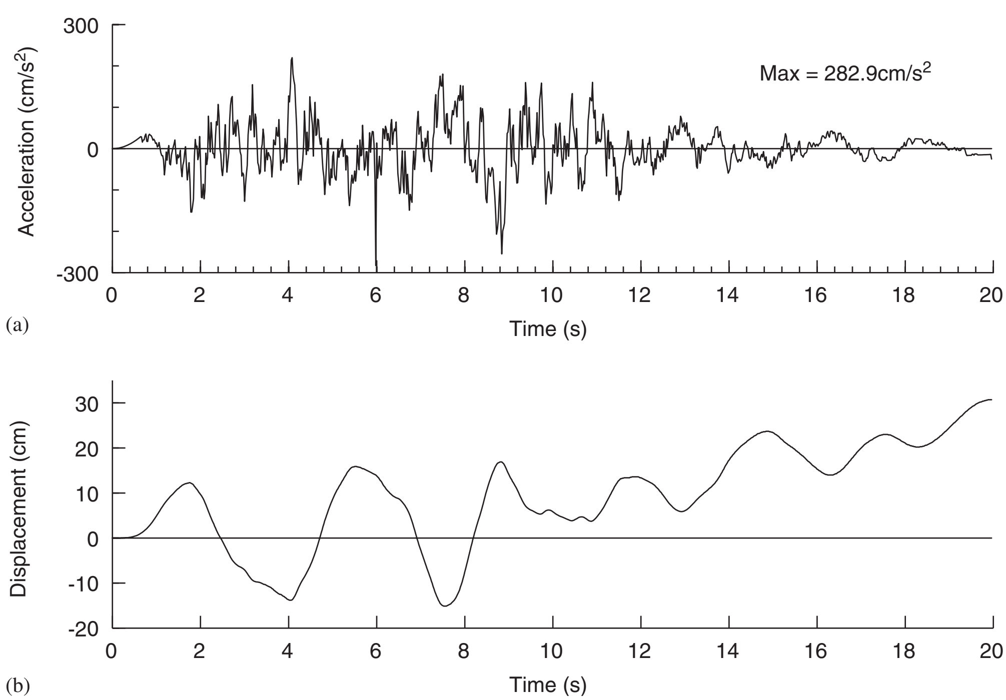

Fig. 3. Acceleration time series used in Example 1: (a) original acceleration, (b) displacement derived by double integration of the acceleration.

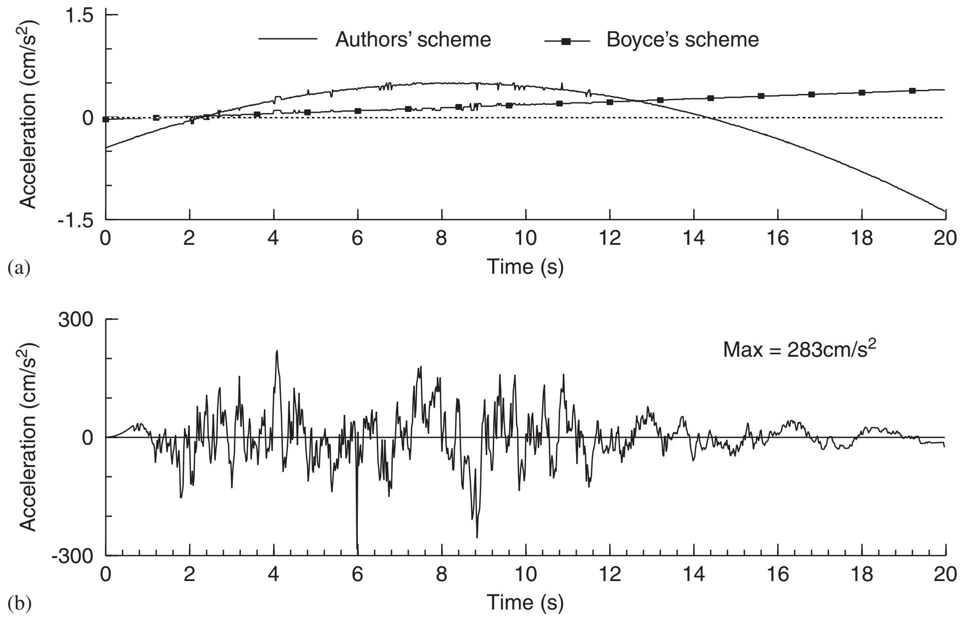

Fig. 4. Baseline correction of the acceleration time series used in Example 1: (a) baseline determined, (b) acceleration time series corrected for the baseline.

affected by this processing. From the standpoint of dynamic soil–structure interaction analysis, use of the velocity and displacement processed in this way as input motions may bring about potential problems of numerical instability.

In seismic soil–structure interaction analysis, the prescribed motion is usually in the form of an observed acceleration record or a synthesized acceleration time series. It is not uncommon that direct integration of the acceleration data provided for a specific soil–structure interaction analysis yields undesirable drifts in the displacement. The purpose of this paper is to suggest a simple, alternative approach to integration of the acceleration data to acquire realistic displacement and velocity excitations for dynamic interaction analysis. The approach involves only processing of the acceleration time series; the displacement and velocity are directly derived from the processed acceleration. The feasibility of the approach is assessed using several examples and comparisons are made between the results obtained using the proposed method and those using other procedures.

2. Simple correction scheme

It is assumed that the baseline of the displacement takes the following form

$$\tilde{u}_{\mathrm{g}}(t) = a_1\, t^4 + a_2\, t^3 + a_3\, t^2 + a_4\, t. \tag{5}$$

The baselines of the velocity and acceleration can then be given as

$$\tilde{\dot{u}}_{\mathrm{g}}(t) = 4a_1\, t^3 + 3a_2\, t^2 + 2a_3\, t + a_4, \tag{6}$$

$$\tilde{\ddot{u}}_{\mathrm{g}}(t) = 12a_1\, t^2 + 6a_2\, t + 2a_3, \tag{7}$$

where a_1–a_4 are four constants to be determined. It is apparent that the drifts in the displacement are assumed here to be the consequence of the parabolic distortions in the acceleration. Noting the common practice in seismic soil–structure analysis that the initial velocity and displacement of the system are assumed to be zero, the coefficient a_4 can be determined. Instead by minimizing the mean square computed velocity, the remaining three coefficients are determined here by minimizing the mean square acceleration as

$$\begin{aligned}\mathrm{Min}&\left\{\sum_{i=1}^{N}\left[(\ddot{u}_{gi} - \tilde{\ddot{u}}_{gi})^2\right]\right\}\\ &= \mathrm{Min}\left\{\sum_{i=1}^{N}\left[(\ddot{u}_{gi} - (12a_1\, t_i^2 + 6a_2\, t_i + 2a_3))^2\right]\right\}.\end{aligned} \tag{8}$$

When the baseline of the acceleration is completely determined, the acceleration is then corrected by subtracting $\tilde{\ddot{u}}_{\mathrm{g}}(t)$ from $\ddot{u}_{\mathrm{g}}(t)$ and the displacement is derived by doubly integrating the corrected acceleration. In many cases the drifts in the displacements can be effectively corrected by using the direct time-domain processing. In the cases where the undesirable long-period fluctuations still occur, a windowed filter is designed for further processing the baseline-corrected acceleration data in the frequency domain.

Referring to Fig. 2, the filter is expressed as

$$\beta(T) = \begin{cases} 1 & 0 \leqslant T \leqslant T_0, \\ \mathrm{e}^{-(T-T_0)/a} & T \geqslant T_0, \end{cases} \tag{9}$$

where a and T_0 are two parameters that can be determined by using two key points A and B (Fig. 2). The two points are selected based on the characteristics of the Fourier

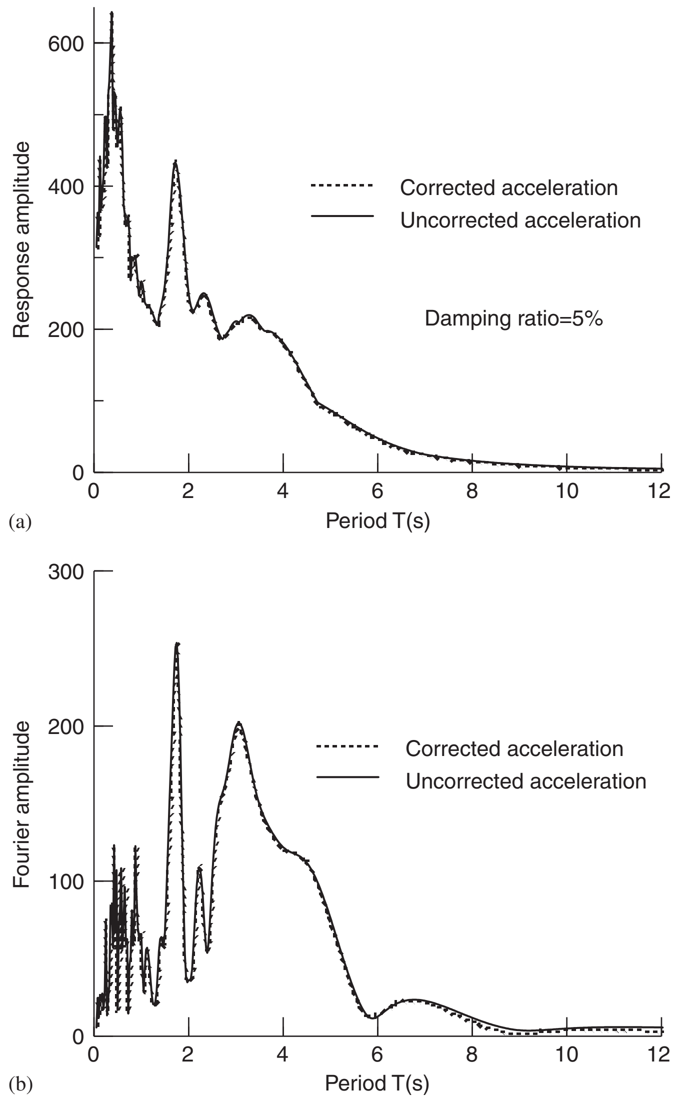

Fig. 5. Effect of baseline correction on frequency characteristics: (a) response spectra, (b) Fourier spectra.

spectra of the displacement time series derived by integrating the uncorrected and baseline-corrected acceleration data. Details will be discussed in the following numerical examples.

3. Numerical examples

3.1. Example 1

Shown in Fig. 3a is an acceleration time series synthesized for seismic soil–structure interaction analysis. The displacement–time series derived by doubly integrating the acceleration is presented in Fig. 3b, showing an unreasonable offset. The baseline of the acceleration can be determined using the proposed scheme as (units: cm/s^2)

$$\tilde{\ddot{u}}_g(t) = -0.01371t^2 + 0.280t - 0.4538. \quad (10)$$

Figs. 4(a) and (b) show the removed non-zero acceleration mean curve and the baseline-corrected acceleration time series, respectively. It can be seen that the change in the time series of the acceleration caused by the baseline correction is negligible. On the other hand, the influence of the baseline correction on the frequency characteristics is also minor, as indicated by Fig. 5, where the response spectra and Fourier amplitude spectra of the uncorrected and corrected acceleration time series are compared.

Fig. 6a shows the baseline of the velocity determined using the proposed scheme along with the velocity derived by integrating the uncorrected acceleration. For purposes of comparison, the baseline determined using the procedure of Boyce [11] is included in the same graph. In Fig. 6b the baseline of the displacement determined using the proposed scheme is presented together with that determined using Boyce's scheme. A linear baseline that is often used to directly correct the displacement is included as well for comparison.

Fig. 7 shows the displacement time series resulted from double integration of the acceleration corrected using the proposed scheme along with the displacement determined using Boyce's scheme and the displacement derived by directly applying a linear baseline correction to the displacement given in Fig. 6b. Generally, the results indicate that the proposed scheme is simple and effective.

3.2. Example 2

In this example an acceleration record obtained during the 1967 Koyna, India earthquake is employed (Fig. 8a). This acceleration record has been widely employed in seismic analysis of concrete dams. Using the proposed scheme, the baseline of the acceleration can be determined as (units: cm/s^2):

$$\tilde{\ddot{u}}_g(t) = 0.03409t^2 - 0.363t + 0.8089. \quad (11)$$

The acceleration time series corrected for the above baseline is shown in Fig. 8b, indicating again that there is almost no change in the time series introduced by the correction. However, double integration of the uncorrected and baseline-corrected acceleration yields significantly different displacements, as shown in Fig. 9. The offsets in the displacement produced by directly integrating the uncorrected acceleration can be effectively removed by using the proposed scheme. For purposes of comparison, the displacements determined using Boyce's procedure and using the approach of Trujillo and Carter [12] are included in Fig. 9. The five weighting factors involved in Trujillo's scheme are assumed to have the same values as in their original paper.

While it suggests the effectiveness of the proposed correction scheme, Fig. 9 implies that some long-period fluctuation appears in the derived displacement and it cannot be removed by the baseline correction alone. To have a better view, a FFT analysis is performed of the displacements derived from the baseline-corrected acceleration and uncorrected acceleration, as shown in Fig. 10. It is noted that a peak appears at a long period, 7.5 s, in the

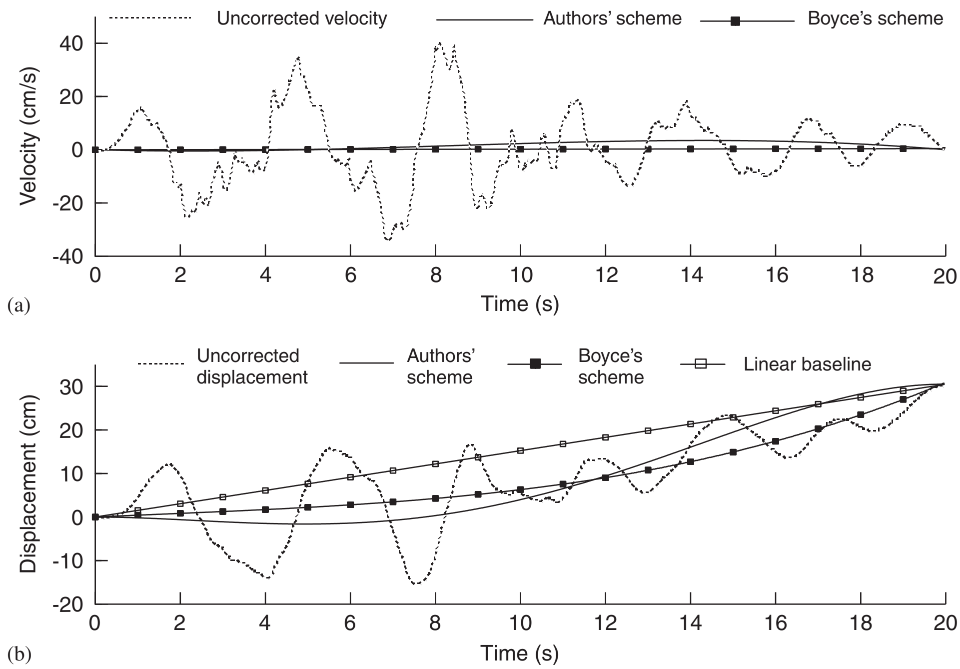

Fig. 6. Comparison of baselines determined using different schemes: (a) baselines of velocity, (b) baselines of displacement.

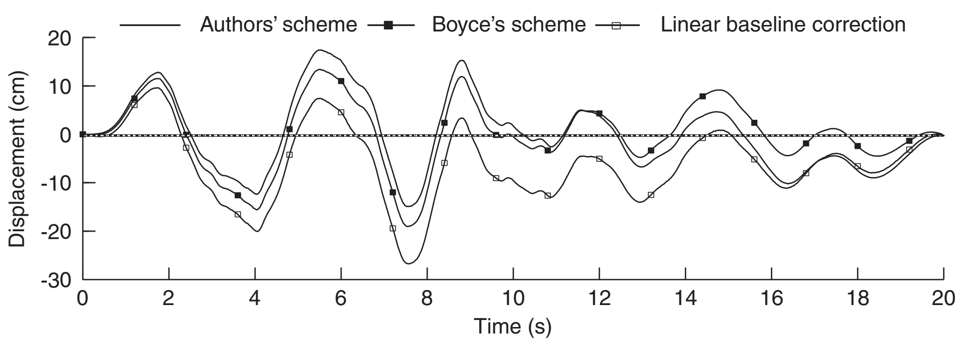

Fig. 7. Displacement time series determined using different correction schemes.

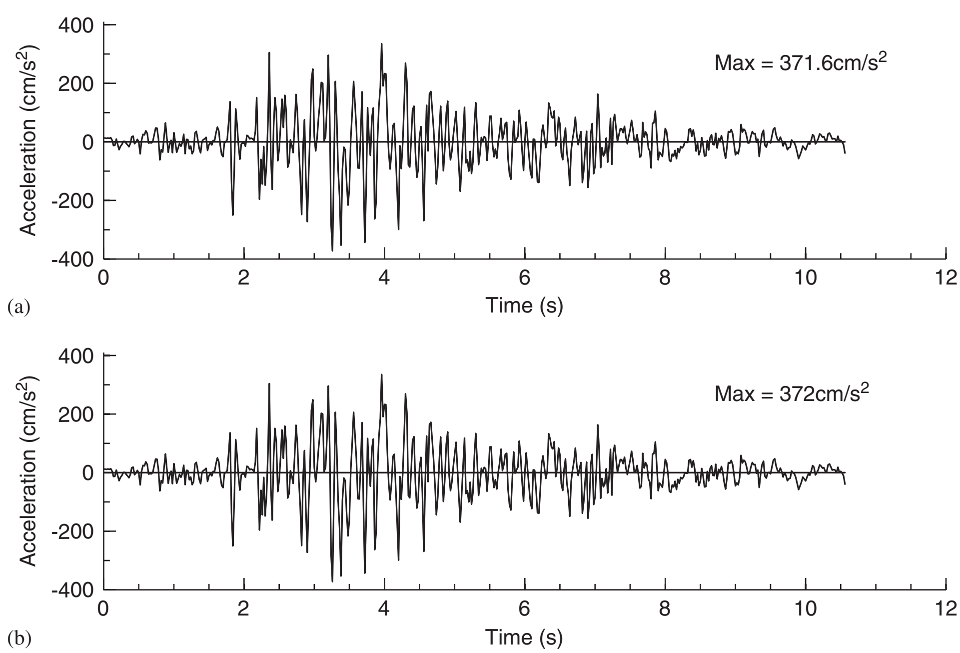

Fig. 8. Acceleration time series used in Example 2: (a) original acceleration, (b) baseline-corrected acceleration.

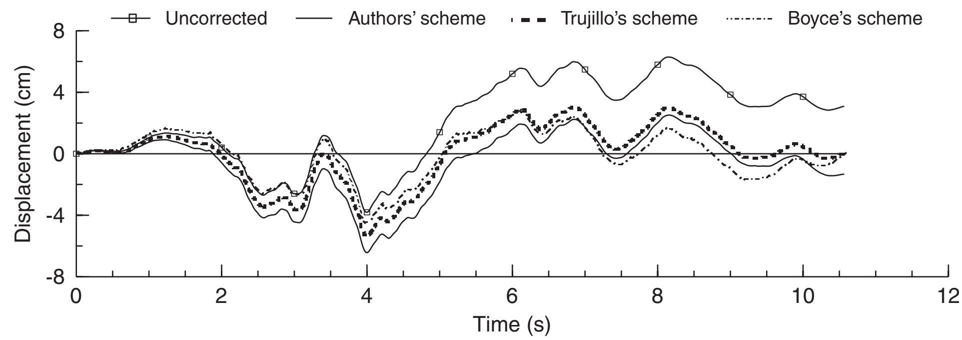

Fig. 9. Displacements determined using different correction schemes for the acceleration time series used in Example 2.

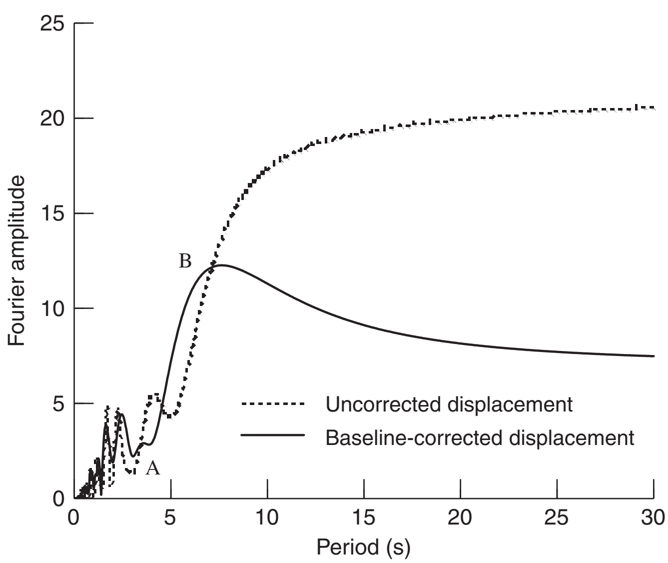

Fig. 10. Fourier spectra of the displacement time series derived from uncorrected and baseline-corrected accelerations.

Fourier amplitude spectrum of the displacement derived from the baseline-corrected acceleration. To further remove the long-period fluctuation, the windowed filter as expressed in Eq. (9) is employed.

Referring to Figs. 2 and 10, the key point B is selected at the peak period, i.e. $T_1 = 7.5$ s, and the key point A is taken at the period of 4 s when a significant increase in the Fourier amplitude initiates, i.e. $T_0 = 4$ s. The parameter a can then be calculated from Eq. (9) given the value of the decay rate at point B. For this example, a typical value of the decay rate, 0.5, is assumed, and the parameter a for the filter is determined to be 5.05.

Fig. 11 shows the displacement derived by doubly integrating the baseline-corrected and filtered acceleration time series together with the displacement derived from the baseline-corrected but unfiltered acceleration. It can be seen that the long-period fluctuation in the displacement–time series can be controlled by using the filter and the effectiveness of filtering is related to the decay rate $\beta(T_1)$. While theoretically a suitable value of $\beta(T_1)$ can be determined by an iterative process until the results are satisfactory, in practical applications it can be decided conveniently by the trial-and-error process.

3.3. Example 3

In this example an acceleration record of long duration as shown in Fig. 1 is used. With the proposed scheme the baseline of the record is determined as (units: cm/s^2)

$$\tilde{\ddot{u}}_g(t) = -0.0003665t^2 + 0.02598t - 0.3415. \quad (12)$$

Fig. 12 shows the displacement time series derived by integrating the baseline-corrected and filtered acceleration record. The two parameters for the filter are determined as $a = 15.79$, $T_0 = 8.2$ s and the decay rate $\beta(T_1)$ is taken as 0.1. The displacements derived using essentially the method of Trifunac [5] and the procedure of Trujillo and Carter [12] are presented in Fig. 12 as well for purposes of comparison. It can be seen that the proposed approach can yield a reasonable displacement even for long duration accelerations.

4. Effects on dynamic response analysis

A simplified model of the dynamic response of a building foundation to earthquake excitation [14] is illustrated in Fig. 13. The horizontal girder in this frame is assumed to be rigid and to include all the moving mass of the structure. The vertical columns are assumed to be weightless and the resistance to girder movement provided by each column is represented by its spring constant, $k/2$; the damper c provides a velocity-proportional resistance to the motion. The mass thus has a single degree of freedom, $u(t)$, induced by the horizontal earthquake ground motion $u_g(t)$.

The equilibrium of forces for this system can be written as

$$m\ddot{u}^t(t) + c\dot{u}(t) + ku(t) = 0, \quad (13)$$

where $u^t(t) = u_g(t) + u(t)$ represents the total displacement of the mass.

It is apparent that Eq. (13) can also be written in the following form:

$$m\ddot{u}(t) + c\dot{u}(t) + ku(t) = -m\ddot{u}_g(t). \quad (14)$$

An alternative form of the equation of motion can be obtained by expressing Eq. (13) in terms of $u^t(t)$ and its

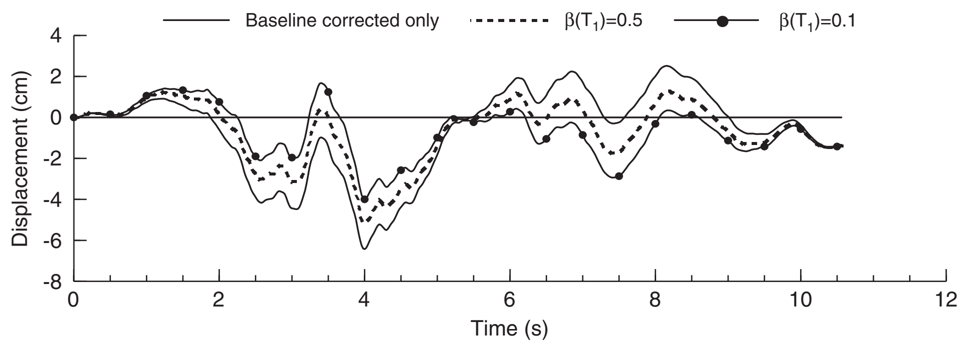

Fig. 11. Displacement time series derived by integrating baseline-corrected and filtered acceleration.

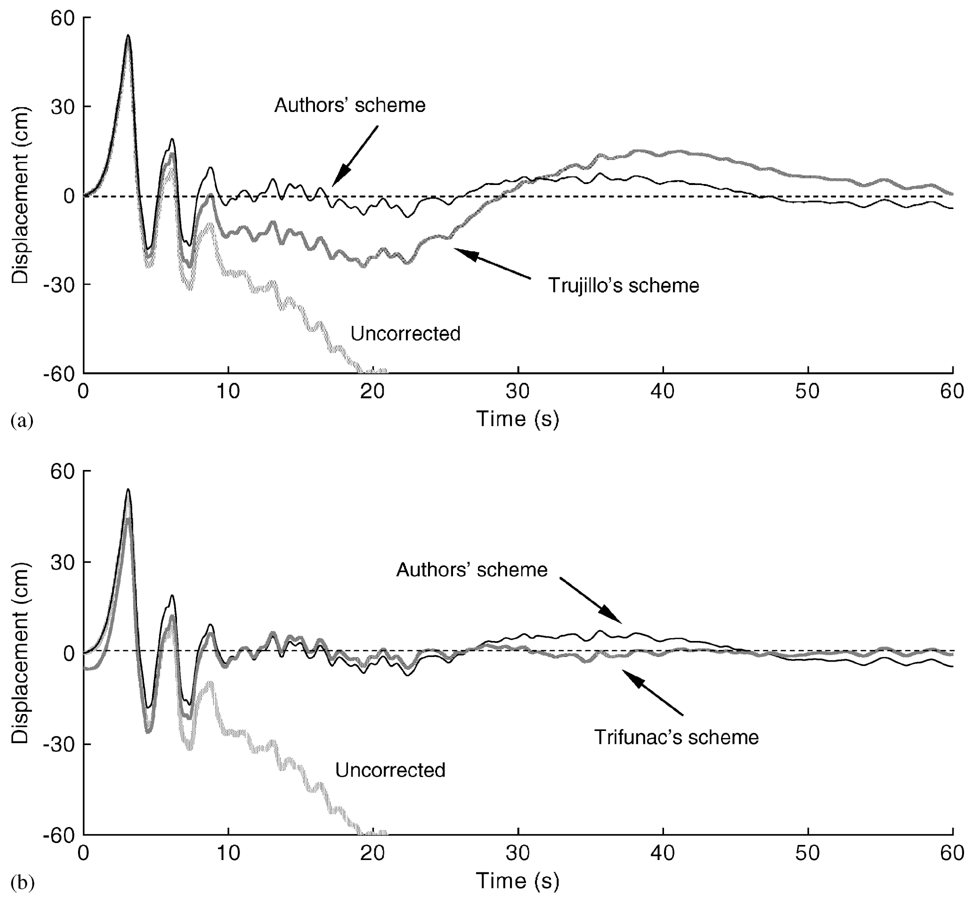

Fig. 12. Displacements derived using different schemes for the acceleration record used in Example 3.

derivatives as

$$m\ddot{u}^t(t) + c\dot{u}^t(t) + ku^t(t) = c\dot{u}_g(t) + ku_g(t). \quad (15)$$

The right-hand side of the above equation represents the effective loading on the structure and depends on the velocity and displacement of the earthquake motion. For convenience, it is assumed that $m = 1$ and then $c = 2\xi\omega$ and $k = \omega^2$, where ω is angular frequency and ξ is damping ratio.

Using the acceleration time series in Example 1 as the earthquake excitation, the response of the system in terms of the relative displacement $u(t)$ can be solved with Eq. (14), as shown in Fig. 14. In the computation the frequency ω is assumed to be $15\,\mathrm{s}^{-1}$ and the damping ratio is taken as 5%.

The dynamic response of the system can also be computed using the equation of motion given in Eq. (15). In doing so, the acceleration $\ddot{u}_g(t)$ is baseline-corrected using the proposed scheme and the velocity $\dot{u}_g(t)$ and displacement $u_g(t)$ are then derived by integrating the processed acceleration. Fig. 14 shows the response of the system in terms of the relative displacement calculated in this way. The calculated response is in agreement with that calculated using the original acceleration as the input excitation, indicating that use of the displacement and velocity derived from the corrected acceleration can yield reasonable results.

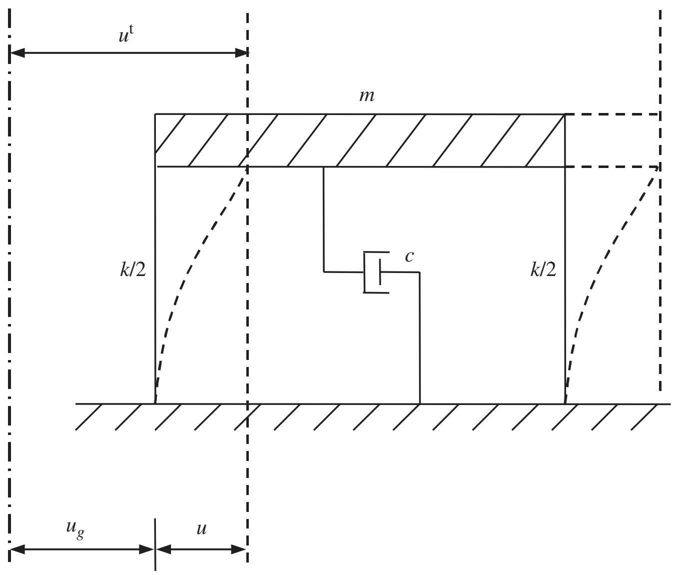

Fig. 13. Simplified model of building foundation under seismic excitation.

On the other hand, it should be noted that for practical, large-scale seismic soil–structure interaction analyses, use of the displacement time series derived from the uncorrected acceleration as the input motion may cause computational instability or unreasonable responses. A schematic illustration is given in Fig. 15, where a long-span bridge is subjected to the passage of seismic wave. Supposing that the time for the seismic wave propagating from pier A to B is Δt, the difference between the displacements at the two piers is then given as $\Delta u_g = u_g(t) - u_g(t + \Delta t)$. If the input displacement contains significant drifts, the displacement difference Δu_g will become unrealistically large, causing the computed response to be inaccurate.

5. Conclusion

Experience has indicated that direct integration of ground acceleration records often causes unrealistic drifts in displacements and velocities. The drifts may have a significant effect on large-scale soil–structure interaction analysis in which the displacement and velocity time series are required as input motions. This paper suggests a simple approach to integration of the acceleration data to acquire reasonable displacement and velocity excitations for the interaction analysis.

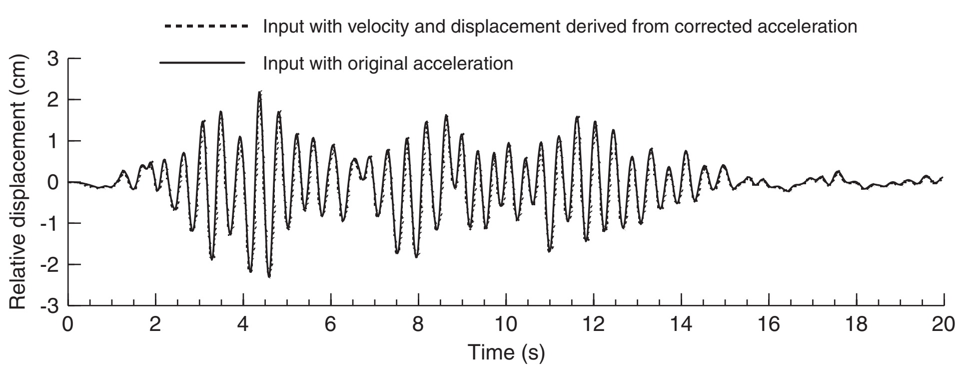

Fig. 14. Displacement response calculated in two different ways.

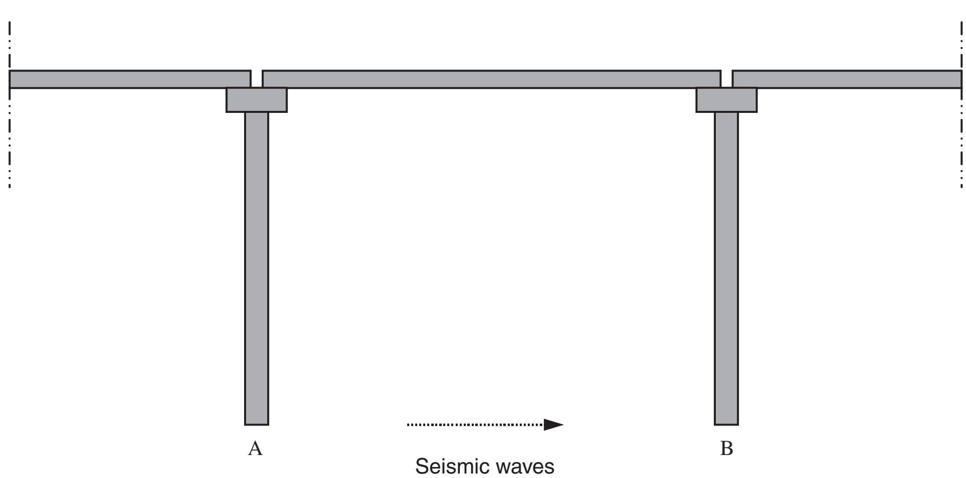

Fig. 15. Bridge subjected to the passage of seismic waves.

In this approach processing is made only of the acceleration time series, firstly in the time domain and then in the frequency domain, and the displacement is directly determined from the processed acceleration. The numerical examples indicate the feasibility and effectiveness of the proposed approach as compared with other more complicated procedures.

Last but not least, it should be mentioned that any permanent ground displacements are eliminated by all correction schemes including the proposed one. To provide reliable estimates of permanent displacements from recorded accelerograms, advanced techniques of recording are needed.

Acknowledgements

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