Saturation Effects of Soils on Ground Motion at Free Surface Due to Incident SV Waves

Jun Yang, M.ASCE[1]

Abstract: A study is presented of saturation effects of subsoil on seismic motions at the free surface of a half space due to an inclined (SV) wave. By treating the soil as a partially water-saturated porous medium that is characterized by its degree of saturation, porosity, permeability, viscosity, and compressibility, a theoretical formulation is developed for the computation of free-surface amplitudes in both the horizontal and vertical components, which are defined as a function of the degree of saturation, the angle of incidence, and the frequency. Numerical results are presented using typical sand properties. It is shown that even a slight decrease of full saturation may lead to a substantial influence on the free-surface amplitudes in both the components and the amplitude ratios between them, and this influence is dependent on the angle of incidence. Significant phase shift between the horizontal and vertical components may also occur due to this slight change in saturation. At small incident angles, partial saturation of subsoil generally may cause a greater vertical-to-horizontal ratio compared with a fully saturated model. It is suggested that one may need to carefully take into account the saturation condition in the interpretation of field observations on seismic ground motions.

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Introduction

Significant amplification of vertical motion was observed by a borehole array at a reclaimed island, Kobe, Japan, during the Kobe earthquake of 1995. A persuasive mechanism that is an alternative to the common explanation related to diffraction effects has been proposed for this observation (Yang and Sato 2000a). Both analytical modeling and numerical simulation have shown that partial saturation of soils may strongly affect vertical ground motion, suggesting the importance of saturation conditions in the interpretation of field observations.

Because of ground water, it has long been recognized that soils may be appropriately regarded as a fully water-saturated porous material. A theory for describing the wave propagation in saturated porous media was successfully established by Biot (1956) and has been widely recognized in geomechanics and geophysics (e.g., Deresiewicz and Skalak 1963; Stoll 1977; Zienkiewicz and Shiomi 1984; Philippacopoulos 1989; Morochnik and Bardet 1996; Yang and Sato 1998). In reality, however, partial saturation of subsoil may occur in certain situations due to fluctuating water tables, flooding, or recharge of groundwater. The situation of partial saturation is a typical case for marine sediments (Sills et al. 1991); also it may exist frequently in offshore sites constructed by land reclamation as evidenced by the Kobe case mentioned above. For this reason, there exists a need to clarify the effects of partial saturation on seismic ground motions, in both the horizontal and vertical components. In particular, it is of interest to study the influence of saturation on the amplitude ratios between the two components, since the results may provide some useful implications to the site evaluation technique generally known as H/V (Nakamura and Ueno 1986), which is based on the interpretation of field observations on both the horizontal and vertical components of microtremors and has recently drawn much attention in engineering practice (e.g., Konno and Ohmachi 1998).

Recently, a preliminary analysis of the seismic reflection at a porous soil interface between two half spaces as affected by the degree of saturation has been conducted (Yang and Sato 2000b). The results showed that the degree of saturation indeed may cause a significant influence on the reflected amplitudes at the interface in some situations. In this paper, a problem corresponding to an inclined (SV) wave incident at the free surface of a homogeneous half space is analyzed. This physical model has been appreciated to be of fundamental importance (Knopoff et al. 1957; Deresiewicz and Rice 1962; Chen et al. 1981; Sharma and Gogna 1991). The objective of this study is to clarify (1) how the degree of saturation affects the free-surface amplitudes in both the horizontal and vertical components; (2) what is the performance of the amplitude ratios between the two components as affected by the saturation; and (3) how the degree of saturation influences the oscillation behavior of soil particles at the free surface. The understanding that is gained is of interest not only at the theoretical level but also in the engineering applications as mentioned previously.

In order to account for the effect of saturation, in the present study, the soil is more appropriately yet simply modeled, based on the concept of homogeneous pore fluid and Biot's theory of two-phase porous media, as a partially water-saturated porous medium that is characterized by its degree of saturation, porosity, perme-

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[1] Alexander von Humboldt Fellow, Institute of Geotechnical Engineering, Technical Univ. of Berlin, Gustav-Mayer-Allee 25, D-13355 Berlin; formerly, JSCE Sr. Researcher, Disaster Prevention Research Institute, Kyoto Univ., Kyoto 611-0011, Japan. E-mail: jun.yang@grundbau.tu-berlin.de

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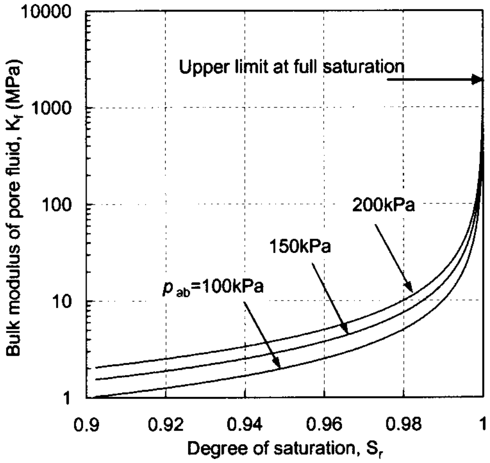

Fig. 1. Influence of saturation on bulk modulus of pore fluid

ability, viscosity, and compressibility of solid and fluid constituents. A theoretical formulation is developed for the computation of displacement amplitudes in both the horizontal and vertical components, which are defined as a function of the degree of saturation, the angle of incidence, and the frequency. Numerical results are presented and discussed using typical sand properties.

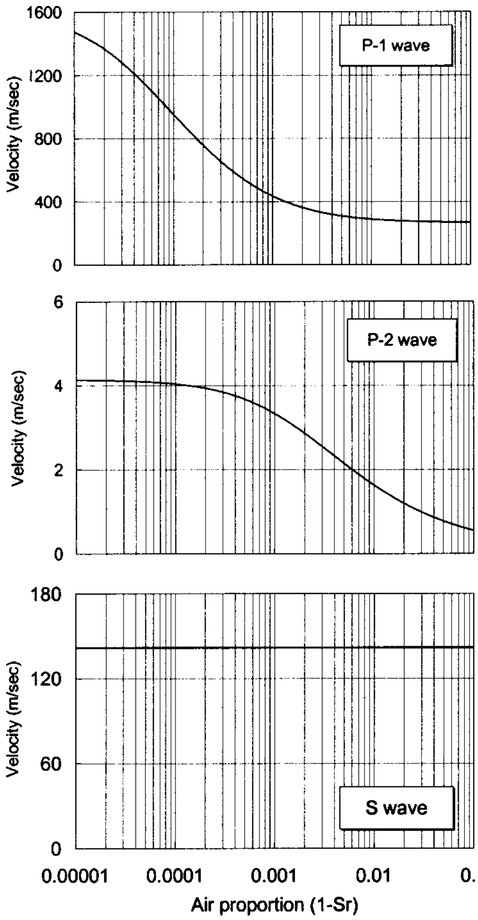

Fig. 2. Effect of saturation on velocities of compression and shear waves in typical sand at frequency of 1 Hz

Table 1. Properties of Typical Sand Used in Computation

Quantity	Notation	Value
Bulk modulus of solid skeleton (MPa)	K_b	86.7
Bulk modulus of solid grains (GPa)	K_s	36
Bulk modulus of water (GPa)	K_w	2.2
Lame constant of solid skeleton (MPa)	μ	40
Permeability (m^2)	k	10^{-11}
Fluid viscosity (Ns/m^2)	η	10^{-3}
Density of grains (kg/m^3)	ρ_s	2650
Porosity	n	0.4

Theoretical Formulation

Apparently, one key difference between the fully water-saturated soil and partially saturated soil is pore fluid. For the partially saturated soil, pore fluid is a mixture of water and air and the relative proportions of constituent volumes can be characterized by the porosity n and the degree of saturation S_r as

$$n=\frac{V_v}{V_t} \quad S_r=\frac{V_w}{V_v} \tag{1}$$

in which V_v and V_w = volumes of pores and pore water, respectively and V_t = total volume. One typical case of partial saturation is when the degree of saturation is sufficiently high (e.g., higher than 90%) so that the air is embedded in pore water in the form of bubbles. For this special case, the concept of homogeneous pore fluid may apply to Biot's theory of two-phase porous media (Biot 1956), and the bulk modulus of the homogeneous fluid K_f can be approximately expressed in terms of the degree of saturation as (Verruijt 1969)

$$K_f=\frac{1}{\dfrac{1}{K_w}+\dfrac{1-S_r}{p_{\mathrm{ab}}}} \tag{2}$$

in which K_w = bulk modulus of pore water and p_{ab} = absolute pore pressure. As shown in Fig. 1, even a very small amount of air in soil can drastically reduce the bulk modulus of pore fluid and this influence is dependent on the absolute pore pressure. Introducing Eq. (2) into Biot's theory, the governing equations can be given in terms of displacements as

$$\mu\nabla^2\mathbf{u}+(\lambda+\alpha^2M+\mu)\nabla e+\alpha M\nabla\zeta=\rho\frac{\partial^2\mathbf{u}}{\partial t^2}+\rho_f\frac{\partial^2\mathbf{w}}{\partial t^2} \tag{3}$$

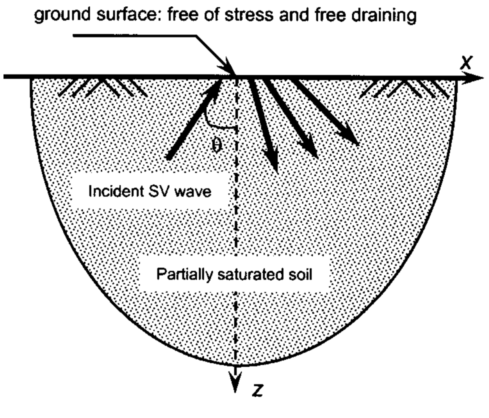

Fig. 3. Physical model considered

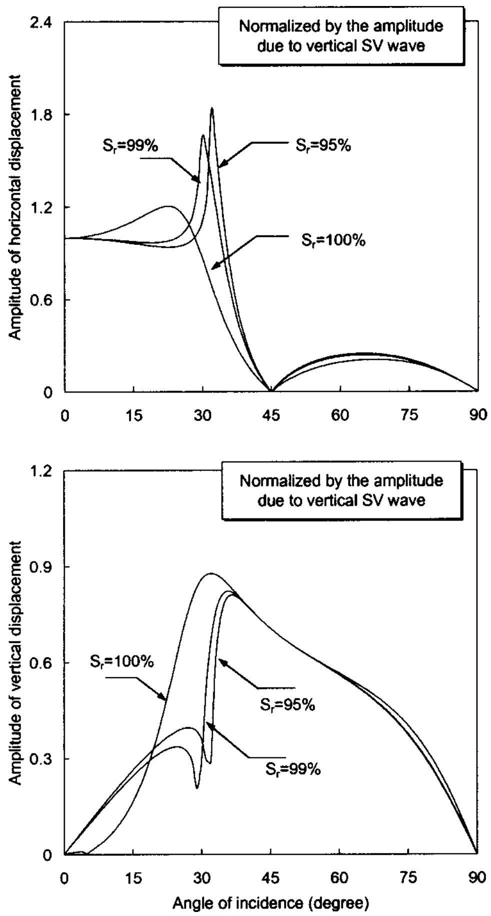

Fig. 4. Effect of saturation on displacement amplitudes in horizontal and vertical components at frequency of 1 Hz

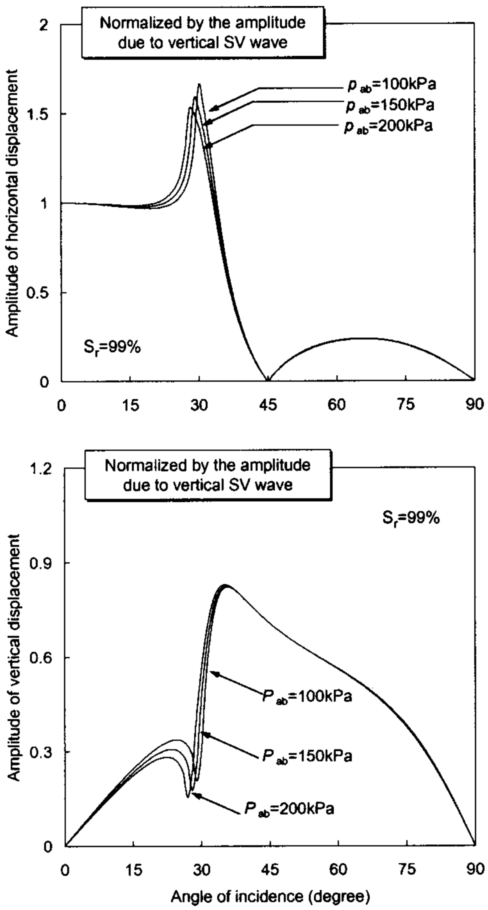

Fig. 6. Influence of absolute pore pressure on free-surface amplitudes at frequency of 1 Hz

$$\alpha M\nabla e - M\nabla\zeta = \rho_f\frac{\partial^2 \mathbf{u}}{\partial t^2} + \frac{\rho_f}{n}\frac{\partial^2\mathbf{w}}{\partial t^2} + \frac{\eta}{k}\frac{\partial \mathbf{w}}{\partial t} \qquad (4)$$

where **u** and **w** = respectively, the displacement vectors of solid skeleton and pore fluid with respect to solid phase; $e = \mathrm{div}\,\mathbf{u}$ and $\zeta = -\mathrm{div}\,\mathbf{w}$ denote, respectively, the volumetric strain of the solid skeleton and the increment of fluid content; η = fluid viscosity and k = permeability (with units of m^2); ρ = total density and ρ_f = density of pore fluid; λ and μ = Lame constants of solid skeleton; and α and M = parameters accounting for the compressibility of grains and fluid, they can be given as

$$\alpha = 1-\frac{K_b}{K_s} \quad M=\frac{K_s^2}{K_d-K_b} \quad K_d=K_s\left[1+n\left(\frac{K_s}{K_f}-1\right)\right] \qquad (5)$$

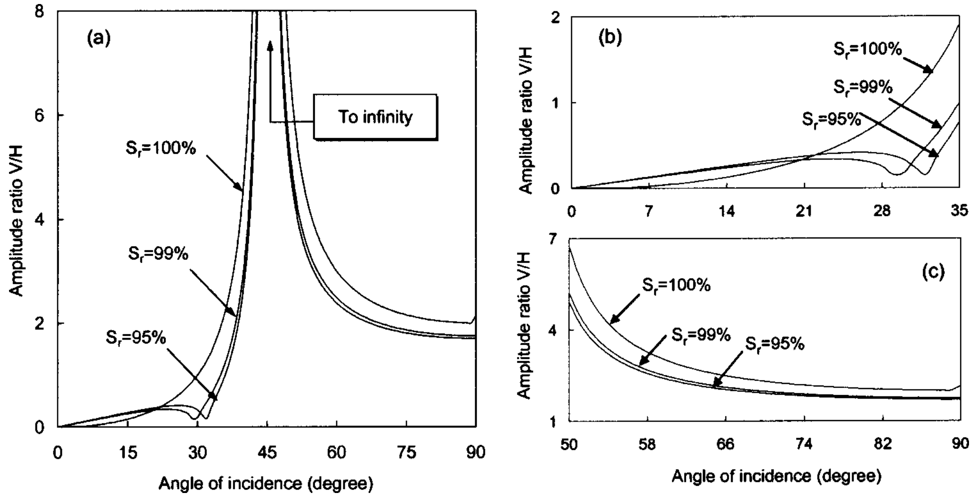

Fig. 5. Effect of saturation on amplitude ratios V/H at frequency of 1 Hz

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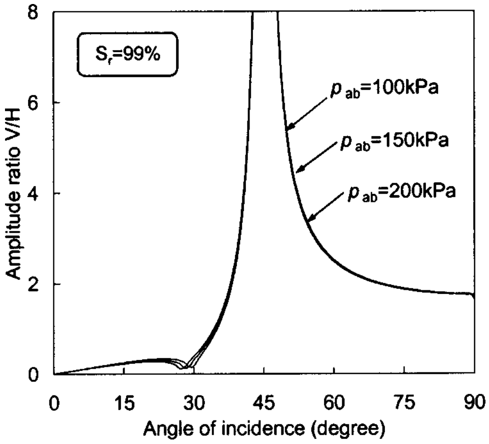

Fig. 7. Influence of absolute pore pressure on amplitude ratios at frequency of 1 Hz

where K_s and K_b = bulk moduli of solid grains and skeleton, respectively and K_f = bulk modulus of pore fluid, being related to the bulk modulus of pore water, absolute fluid pressure, and degree of saturation as described in Eq. (2).

It should be noted that k in the formulation is different from the permeability coefficient k' (m/s) that is usually used in soil mechanics and they are related by

$$k = k' \frac{\eta}{\rho_f g} \tag{6}$$

in which g = gravitation acceleration at which the permeability is measured.

The relationships between stress, pore pressure, and strain entering into the governing equations are as follows:

$$\sigma_{ij} = \lambda e \delta_{ij} + 2\mu \varepsilon_{ij} - \alpha \delta_{ij} p_f \tag{7}$$

$$p_f = M\zeta - \alpha M e \tag{8}$$

$$\varepsilon_{ij} = \tfrac{1}{2}(u_{i,j} + u_{j,i}) \tag{9}$$

where σ_{ij} = total stress; p_f = pore pressure, and δ_{ij} = Kronecker delta.

With the aid of Helmholtz resolution (e.g., Biot 1956; Deresiewicz and Rice 1962), the governing equations can be conveniently solved to yield two compression waves (herein referred to as P-1 and P-2 waves, respectively) and one shear wave (referred to as the S wave). All the three waves are frequency dependent and attenuated, and the influence of saturation on each of them is different. Fig. 2 shows the velocities of the three types of waves for a typical sand as a function of air proportion $(1 - S_r)$, for the frequency of 1 Hz. The properties of the typical sand are given in Table 1. It can be seen that the velocity of the P-1 wave decreases substantially with even a slight decrease below full saturation. The velocity reaches maximum at full saturation, about 1,470 m/s, while it dramatically drops to 355 m/s when the degree of

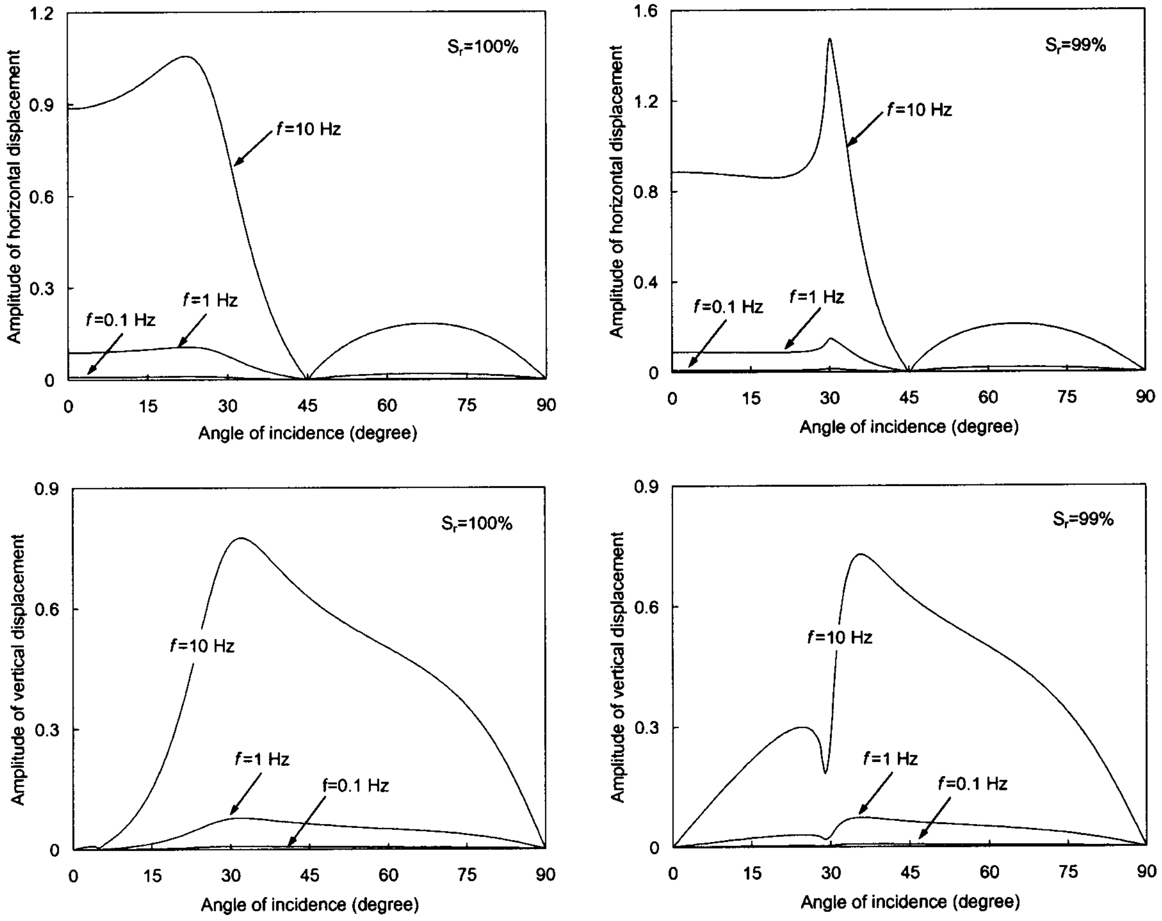

Fig. 8. Angle-dependent displacement amplitudes at three different frequencies

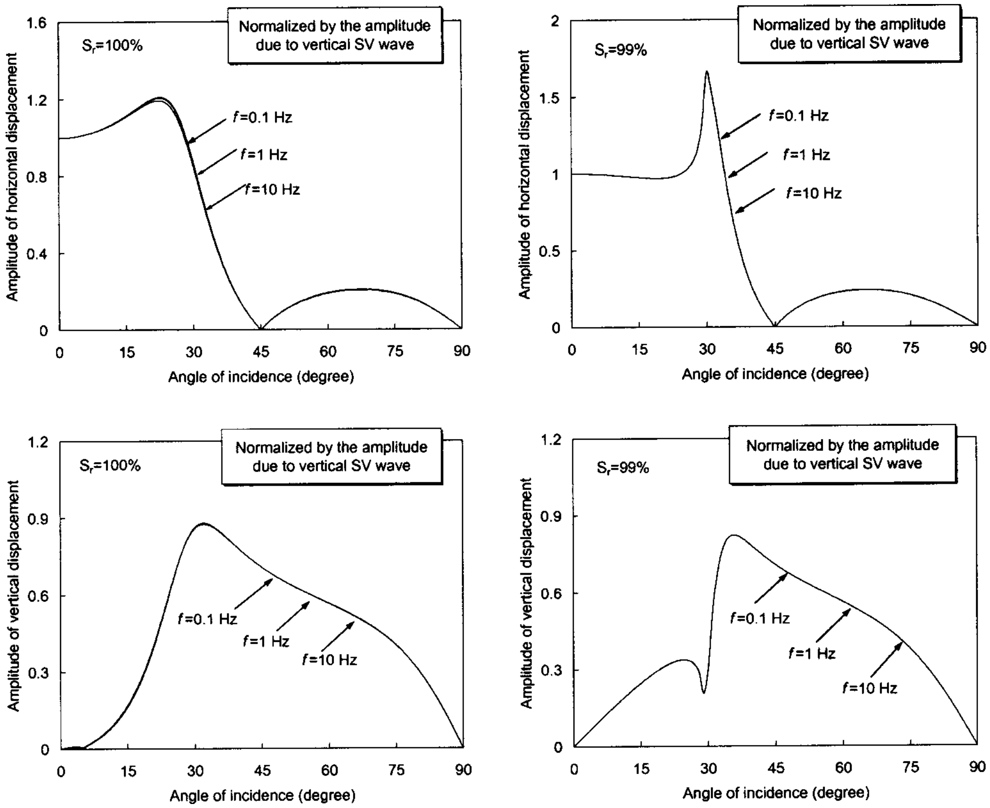

Fig. 9. Normalized displacement amplitudes at three different frequencies

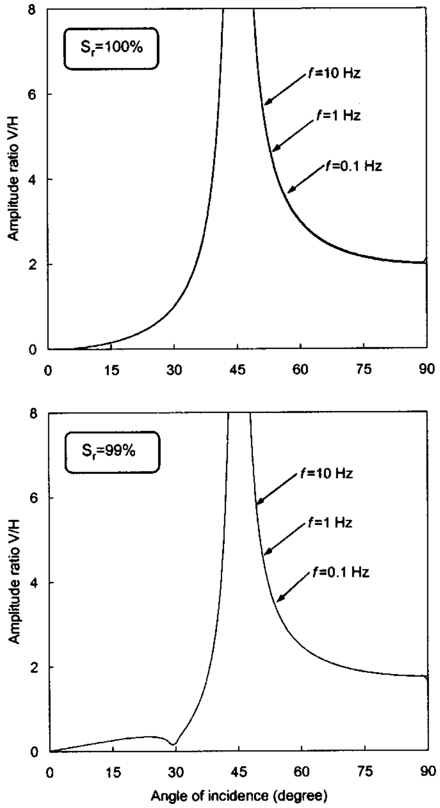

Fig. 10. Angle-dependent amplitude ratios at three different frequencies

saturation is 99.8%. For the S wave, as expected, the influence of saturation is negligible. The velocity of the P-2 wave also reduces with decreasing saturation although it is generally very small at low frequencies due to viscous coupling between solid and fluid phases.

The boundary-value problem under consideration is shown in Fig. 3. An inclined SV wave with incident angle θ and angular frequency ω, is incident to the free surface of a half space that is made up of partially saturated soil. As a consequence, three reflected waves are generated. Referring to the coordinate system, the wave fields in the region of $z>0$ can be conveniently expressed in terms of potentials as follows:
SV waves:

$$\psi_s=[B_{s1}\exp(iq_3z)+B_{s2}\exp(-iq_3z)]\Omega(x,t) \tag{10}$$

$$\psi_f=[\delta_3B_{s1}\exp(iq_3z)+\delta_3B_{s2}\exp(-iq_3z)]\Omega(x,t) \tag{11}$$

P waves:

$$\varphi_s=[A_{s1}\exp(-iq_1z)+A_{s2}\exp(-iq_2z)]\Omega(x,t) \tag{12}$$

$$\varphi_f=[\delta_1A_{s1}\exp(-iq_1z)+\delta_2A_{s2}\exp(-iq_2z)]\Omega(x,t) \tag{13}$$

where $\Omega(x,t)=\exp[i(\omega t-px)]$ and $i=\sqrt{-1}$; δ_1, δ_2, and δ_3 $=$ amplitude ratios of potentials related to solid and fluid phases in porous soil, which can be determined as (Yang 1999)

$$\delta_1=\frac{(\lambda+2\mu+\alpha^2M)l_1^2-\rho\omega^2}{\rho_f\omega^2-\alpha Ml_1^2} \tag{14}$$

$$\delta_2=\frac{(\lambda+2\mu+\alpha^2M)l_2^2-\rho\omega^2}{\rho_f\omega^2-\alpha Ml_2^2} \tag{15}$$

$$\delta_3 = \frac{\mu l_s^2 - \rho\omega^2}{\rho_f \omega^2} \tag{16}$$

in which $l_1, l_2 =$ complex wave numbers of the P-1 and P-2 waves in porous soil, respectively, and $l_s =$ complex wave number of the SV wave.

The ground surface requires the traction to vanish at $z=0$. In addition, the surface is conventionally assumed to be free draining, which leads to the vanishing of the pore pressure. By enforcing these boundary conditions one may obtain the reflection and transmission coefficients for the potentials as a function of the degree of saturation, incident angle, and frequency. The horizontal and vertical displacements at the surface can subsequently be given as

$$u_x = i(q_3 B_{s2} - q_3 - pA_{s1} - pA_{s2})\Omega(x,t) \tag{17}$$

$$u_z = -i(pB_{s2} + p + q_1 A_{s1} + q_2 A_{s2})\Omega(x,t) \tag{18}$$

Numerical Results

Effect of Saturation on Angle-Dependent Displacement Amplitudes

The properties of a typical sand as given in Table 1 are employed in the numerical examples. Fig. 4 shows the free-surface amplitudes of the horizontal and vertical components of displacements as a function of incident angle. The frequency in the computation is taken as 1 Hz and the absolute pore pressure p_{ab} is taken as the atmospheric pressure, 100 kPa. Three degrees of saturation are included to show the influence of saturation. In order to simultaneously illustrate the influence of incident angle, both the horizontal and vertical amplitudes are normalized by the amplitude due to a vertically incident SV wave. It is noted that, for all the cases of saturation under consideration, the displacement in the horizontal component vanishes at an angle of 45°. Generally, the amplitude of horizontal displacement due to an inclined SV wave is smaller than that induced by a vertical SV wave when the angle of incidence is greater than about 40°. However, in a narrow range of angles (i.e., between 20 and 35°), a peak forms with a peak value significantly exceeding one for all cases of saturation. A substantial difference exists in this range between the cases of full and partial saturation. Even a slight decrease of complete saturation leads to a sharper peak with a higher-peak value, whereas the vertical component at this point is found to form a downward cusp. In general, partial saturation may cause larger amplitudes in the vertical component but lower amplitudes in the horizontal component at small angles of incidence (below about 19°).

The results for the amplitude ratios between the vertical and horizontal components are shown in Fig. 5 as a function of the angle of incidence. A singularity in the ratios can be observed at the angle of 45°, this is because the horizontal displacement is zero at this point, as shown in Fig. 4. A closer view of the ratios beyond the singularity is given in Figs. 5(b and c) indicating a notable effect of saturation exists and this effect is also angle dependent. For instance, when the SV wave is nearly normally incident at the surface (with an incident angle of 3°), the vertical-to-horizontal ratio for $S_r = 99\%$ can be as large as seven times the ratio for $S_r = 100\%$.

To examine the influence of absolute pore pressure on the results, Fig. 6 illustrates the free-surface amplitudes in both the horizontal and vertical components at $S_r = 99\%$ for three absolute pore pressures (100, 150, and 200 kPa, respectively). The frequency of the incident wave is also assumed as 1 Hz. The computed amplitude ratios for these three absolute pore pressures are shown in Fig. 7. In general, the influence of the absolute pore pressure is found to be small. A slight discrepancy only exists in the horizontal and vertical amplitudes in a narrow range of angles between 20 and 40°.

The results shown previously are obtained at a specified frequency of 1 Hz. In order to take into account the effect of frequency on the results, three typical frequencies (0.1, 1, and 10 Hz) that may represent the low, intermediate, and high frequencies, respectively, are used in the computation. Fig. 8 shows the two-dimensional amplitudes at the free surface for $S_r = 100$ and 99% for the three frequencies. It is to be noted that the amplitudes are not normalized by the amplitude due to a vertical SV wave. Obviously, the increase of frequency may cause larger amplitudes in both the horizontal and vertical components almost over the whole range of angles. However, if these amplitudes are normalized by the amplitude induced by a vertical SV wave, the effect of frequency will disappear, as shown in Fig. 9. This can also be observed in the angle-dependent amplitude ratios between the vertical and horizontal components as shown in Fig. 10.

Effect of Saturation on Oscillation Behavior of Soil Particles

The understanding of oscillation behavior of soil particles is useful in the interpretation of seismic motion data. By treating the soil as an ordinary solid medium, Meissner (1965) discussed the oscillation of soil particles and showed the phase shift between the horizontal and vertical displacements. How the particle motion is affected by the saturation of subsoil is of interest and deserves an investigation. Fig. 11 shows the particle displacements at the position $x=0$ in the horizontal and vertical components in the situations of full and partial saturation, which are induced by a vertical SV wave of 1 Hz. The resultant particle motions for these two cases of saturation are also depicted in the same figure for a better comparison. Corresponding results for the angles of incidence of 15, 30, and 45° are shown in Figs. 12, 13, and 14, respectively.

Fig. 11 indicates that, for a normal incident SV wave, there is no vertical motion in either the fully or partially saturated case, and the particle motion is thus in the horizontal direction only. When the SV wave is incident at an angle of 15° (Fig. 12), it is found that the horizontal and vertical components are almost out of phase by 90° in the case of full saturation. As a consequence, the particle motion is elliptical. However, quite a different feature is noticed when the subsoil is not completely saturated, even if the degree is very slight: the two components of motion become almost in phase, resulting in the particle motion in a linear manner. At this angle, the amplitude in the horizontal component is also found to be much greater than that in the vertical component for both cases of saturation.

The results for the angle of 30° (Fig. 13) show that the horizontal and vertical motions are still out of phase at full saturation and thus the particle motion is elliptical too. But different from the behavior for an angle of 15°, the long axis of the ellipse is now shifted to the vertical direction. The amplitude in the horizontal component is almost the same with that in the vertical component. Again, it is interesting to note that, when the subsoil is not fully saturated, the amplitude of horizontal displacements becomes much larger than that in the vertical component. The two components of motion are still out of phase by 90° but the long

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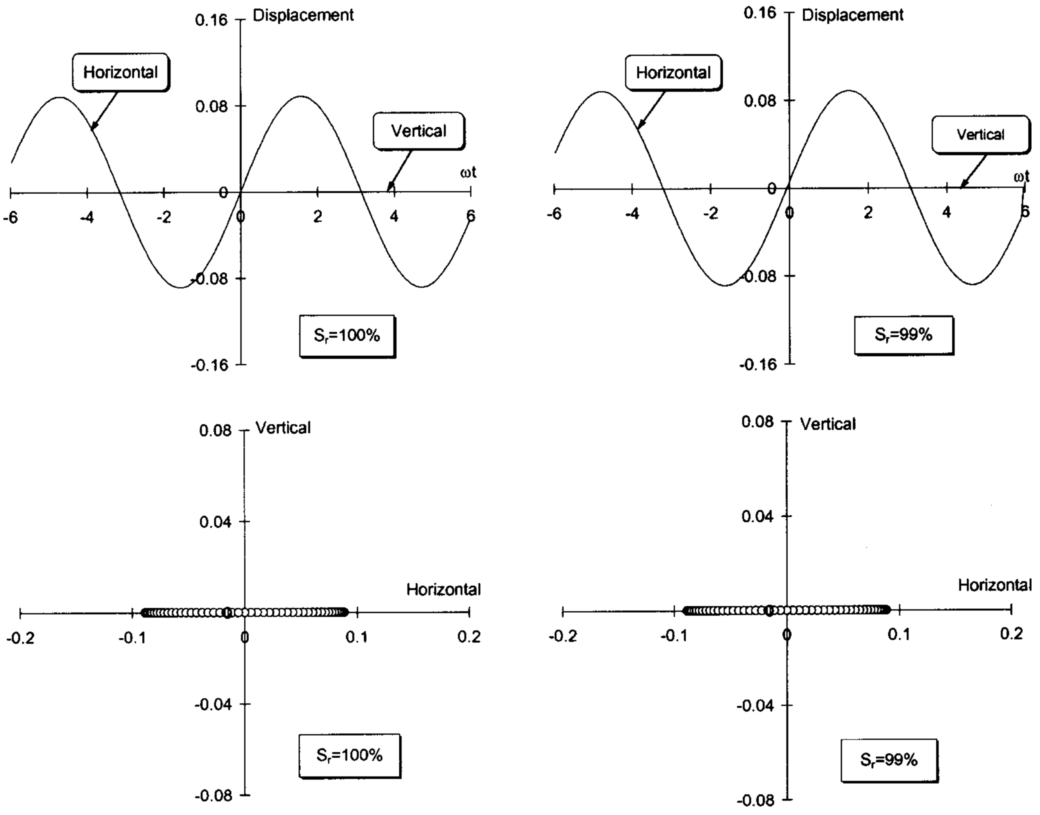

Fig. 11. Effect of saturation on particle motion at free surface (normal incidence)

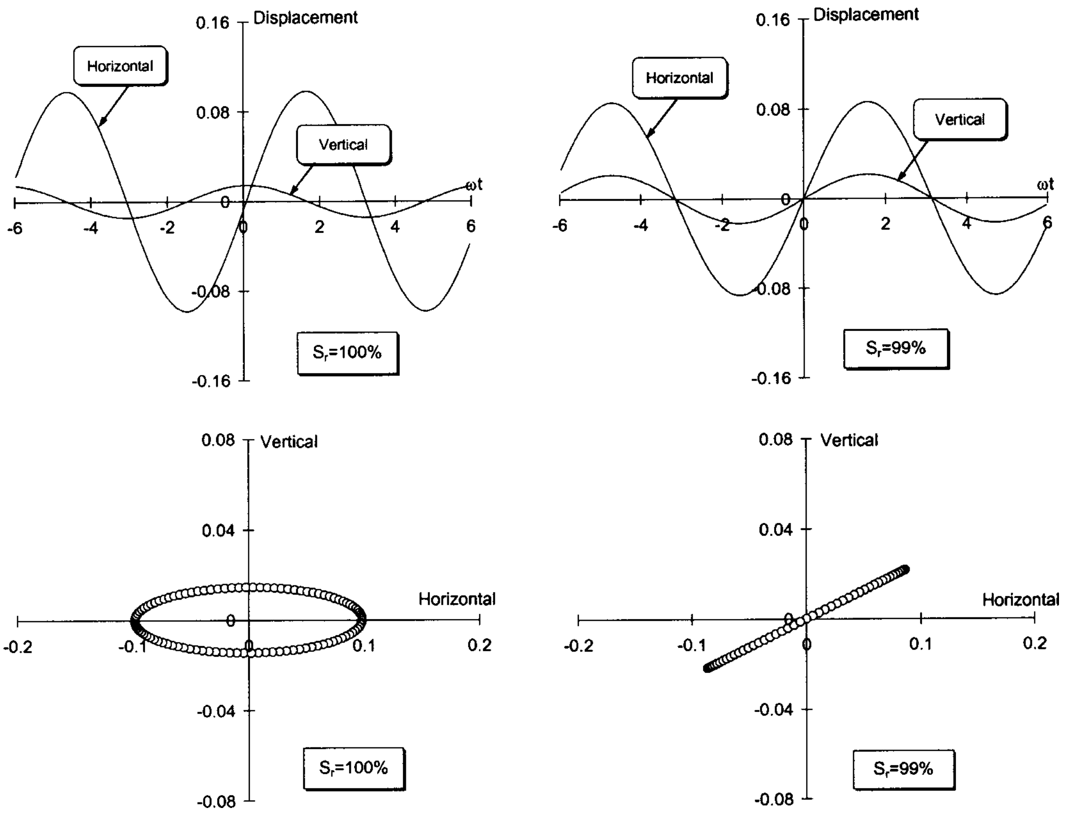

Fig. 12. Effect of saturation on particle motion at free surface (incident angle = 15°)

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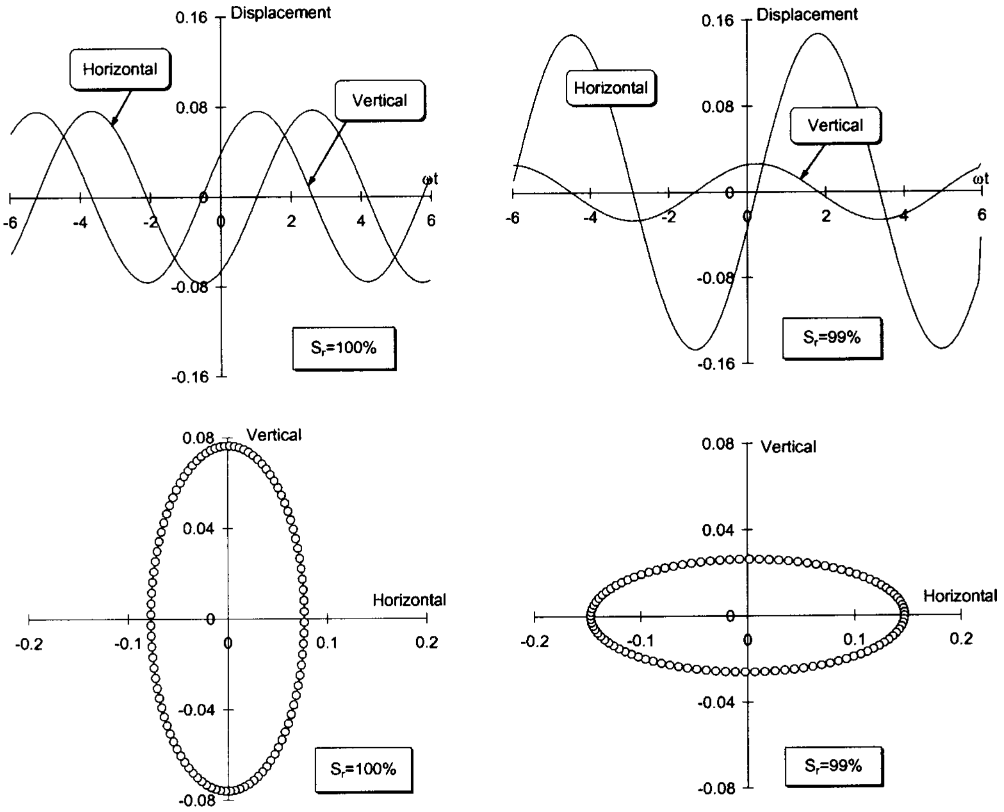

Fig. 13. Effect of saturation on particle motion at free surface (incident angle = 30°)

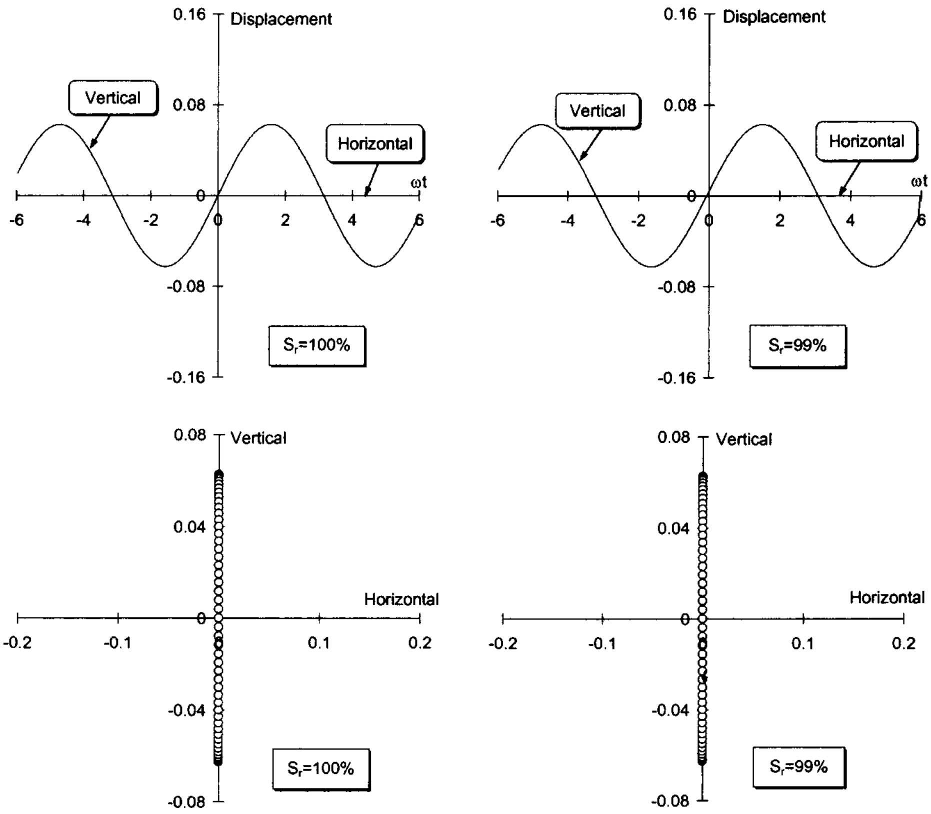

Fig. 14. Effect of saturation on particle motion at free surface (incident angle = 45°)

axis of the ellipse is shifted to the horizontal direction. At an incident angle of 45° (Fig. 14), the particle motion is found to be along the vertical direction only in both cases of saturation; this is reasonable because there is no horizontal component of motion at this angle.

Concluding Remarks

A study has been described of the saturation effect of subsoil on the ground motions at the free surface of a half space induced by an inclined SV wave. Based on the concept of homogeneous pore fluid and Biot's theory, a theoretical formulation was presented for the computation of the two-dimensional free-surface amplitudes that were defined as a function of the degree of saturation, the angle of incidence, and the frequency.

Numerical results show that even a slight decrease from full saturation may have a significant effect on the free-surface amplitudes in both the horizontal and vertical components and the amplitude ratios between the two components, and this influence is dependent on the angle of incidence. The oscillation of soil particles may also be strongly affected by even a slight decrease from full saturation. In general, partial saturation may cause larger vertical-to-horizontal ratios at the free surface for small angles of incidence. The results imply that one may need to carefully take into account the saturation condition in interpreting field observations of ground motions.

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Notation

The following symbols are used in this paper:

e = $\operatorname{div}\mathbf{u}$, volumetric strain of solid skeleton;
i = $\sqrt{-1}$;
K_b = bulk modulus of solid skeleton;
K_f = bulk modulus of pore fluid;
K_s = bulk modulus of solid grains;
K_w = bulk modulus of pore water;
k = permeability (unit: m^2);
k' = permeability coefficient (unit: m/s);
l_s = wave number of S wave;
l_1, l_2 = wave numbers of P-1 and P-2 waves in porous soil;
n = porosity;
p_{ab} = absolute pore pressure;
S_r = degree of saturation;
$\mathbf{u}$ = displacement vector of solid skeleton;
V_t = total volume of unit;
V_v = volume of pores;
V_w = volume of pore water;
$\mathbf{w}$ = displacement vector of pore fluid with respect to solid phase;
α, M = parameters accounting for compressibility of grains and fluid;
$\delta_1, \delta_2, \delta_3$ = amplitude ratios of potentials related to solid and fluid phases;
ζ = $-\operatorname{div}\mathbf{w}$, a measure of change in pore fluid amount;
η = viscosity of pore fluid;
λ, μ = Lame's constants of solid skeleton;
ρ = total density;
ρ_f = density of pore fluid;
φ_s, Ψ_s = potentials associated with solid phase of bulk material;
φ_f, Ψ_f = potentials associated with flow of pore fluid; and
ω = angular frequency.

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