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TECHNICAL NOTE

Shear wave velocity in sand: effect of grain shape

X. LIU* and J. YANG†

Whether the shear wave velocity in sand is dependent on grain shape is a basic question of considerable interest, yet remains open owing to the lack of solid experimental evidence. This technical note presents results from an experimental study where the effect of grain shape was carefully isolated from other factors and the grain shape was accurately measured using laser scanning technology. It is shown that, under otherwise similar conditions, shear wave velocity or the associated small-strain stiffness tends to decrease as sand grains become rounded, and that a fairly good correlation exists between stiffness parameters and a shape index known as overall regularity that accounts for grain shape in a collective manner. The study suggests that the laboratory finding reported in the literature, that a decrease in roundness of sand grains leads to a decrease in shear wave velocity or stiffness, was most likely attributable to varying gradation of tested materials rather than to varying grain shape.

KEYWORDS: dynamics; elasticity; laboratory tests; sands; stiffness

INTRODUCTION

Shear wave velocity (V_s) or the associated small-strain stiffness (G_0) is a soil property that plays a vital role in a variety of geotechnical applications. Numerous studies have been conducted to investigate the characteristics of this property for sands, and it is now well recognised that the stiffness of sand is highly dependent on the packing density (or void ratio) and the effective confining stress. In recent years, there has been growing interest in understanding the potential influence of grain characteristics on V_s or G_0, such as the effect of particle size (Sharifipour *et al*., 2004; Yang & Gu, 2013), the effect of gradation (Wichtmann & Triantafyllidis, 2009; Liu *et al*., 2016) and the effect of fines (Wichtmann *et al*., 2015; Yang & Liu, 2016). As far as the effect of grain shape is concerned, a notable study was by Cho *et al*. (2006), who compiled a database of natural and crushed sands from the literature and quantified shape parameters for these sands using the reference shape chart proposed by Krumbein & Sloss (1963). One of the significant findings of their study was that a decrease in roundness of sand grains leads to a decrease in shear wave velocity or stiffness, as shown in Figs 1(a) and 1(b). Note that the α-factor in the plot represents the shear wave velocity at the confining stress of 1 kPa, while the exponent β reflects the stress dependence of shear wave velocity and has a limited range for natural sands.

It is worth noting, however, that the data in Figs 1(a) and 1(b) cover a range of sands with different size distributions. A careful examination of the database shows that the coefficients of uniformity (C_u) of these sands vary from as low as 1·4 to as high as 5·5. Recalling the laboratory observations that V_s or G_0 depends on grading (Wichtmann & Triantafyllidis, 2009), the same dataset is reinterpreted as a function of the coefficient of uniformity, as shown in Figs 1(c) and 1(d). It is immediately evident that a reasonable correlation also exists between V_s and C_u that V_s tends to decrease with increasing C_u. This trend is consistent with the findings of Wichtmann & Triantafyllidis (2009) and Liu *et al.* (2016), thus raising concerns that the variation of V_s reported by Cho *et al.* (2006) might not be a true reflection of the shape effect, but rather be associated with varying gradation.

In a more recent study, Altuhafi *et al*. (2016) introduced a new shape parameter SAGI to collectively account for aspect ratio, convexity and sphericity of sand grains, and showed for a database of natural sands that the small-strain shear stiffness (G_0), normalised for size and gradation, increases with SAGI. This result implies that sands with rotund particles exhibit lower stiffness, which is not in agreement with the finding of Cho *et al*. (2006) as discussed above. Unfortunately, a lack of strong correlation can be seen between the normalised stiffness and the shape parameter; this is likely attributable to various sources of uncertainty in the database used for the analysis, particularly the uncertainties associated with compiling G_0 data from the various studies in the literature that involved sands with different mineralogy and involved different testing techniques and sample preparation methods. These uncertainties make it difficult to isolate the shape effect, raising the concern that the observed variation of G_0 with particle shape could be due to many other factors.

The foregoing discussion is not to belittle previous research, which, in the authors' opinion, constitutes a valuable contribution towards a better understanding of this important subject, but rather is to indicate that the problem concerned is difficult. In the current context, whether the shear wave velocity or the associated stiffness depends on grain shape remains an open question. This technical note presents an attempt to address this fundamental question through specifically designed laboratory experiments. The novelty of the experimental programme lies in the integration of the three aspects as follows: first, the effect of grain shape was carefully isolated from the effects of grading, size and mineralogy; second, the grain shape of the tested materials was accurately quantified using laser scanning and imaging technology so that the subjectivity associated with the

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* Department of Geology Engineering, Chang'an University, Xi'an, P. R. China; formerly The University of Hong Kong.
† Department of Civil Engineering, The University of Hong Kong, Hong Kong, P. R. China.

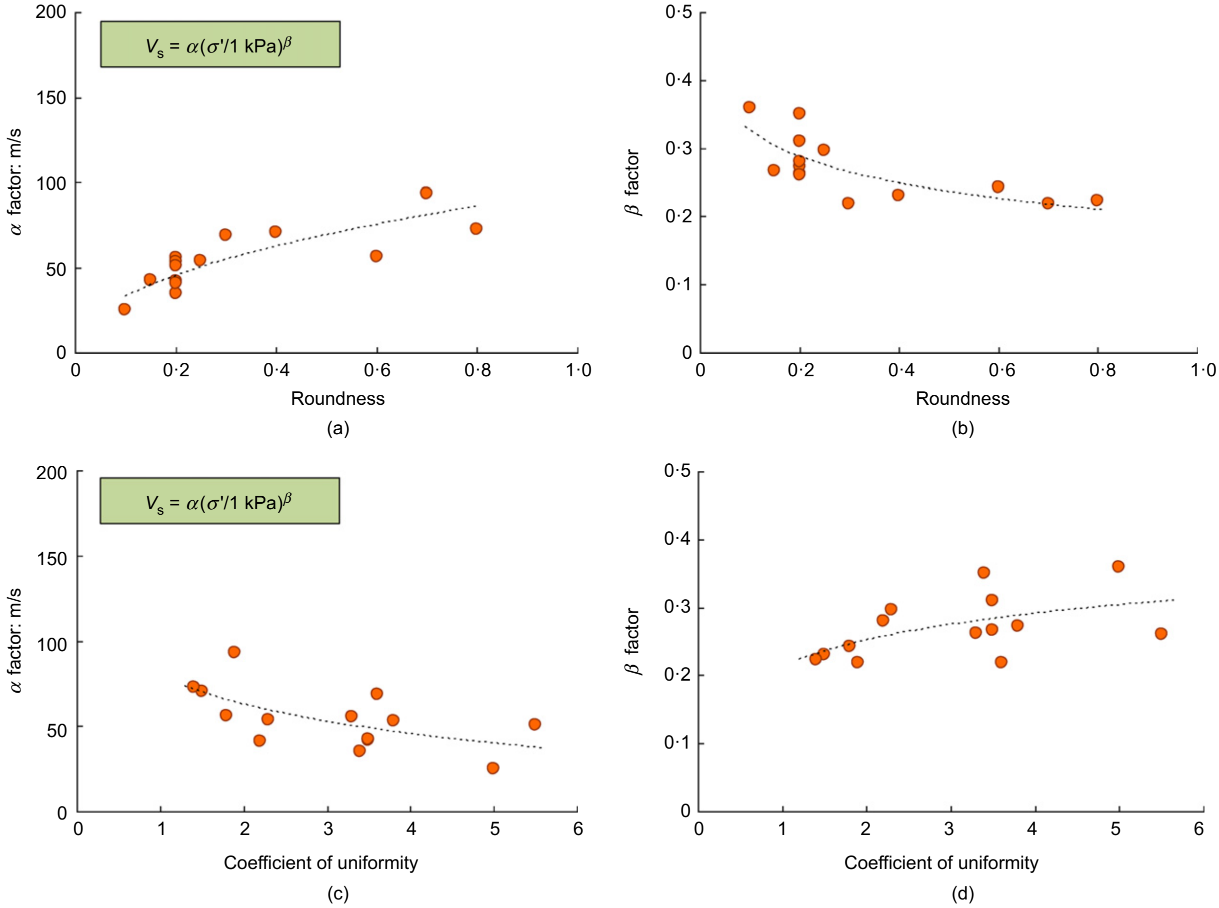

Fig. 1. Observed variation of shear wave velocity in sands: (a), (b) α factor and β factor as a function of roundness (after Cho *et al.*, 2006); (c), (d) α factor and β factor as a function of uniformity coefficient (re-interpreted)

traditional visual inspection method was minimised; and third, a shape index that accounts for grain shape in a more comprehensive manner was used to interpret the data. The main test results are presented along with interpretation and discussion. These results not only provide convincing evidence for the shape effect on V_s and G_0, but can also serve as a useful reference for validation and calibration of numerical and theoretical developments.

TESTING PROGRAMME

Four quartz sands, namely, Ottawa sand, Toyoura sand, Fujian A sand and Fujian B sand, were used in the laboratory experiments. To ensure identical grading, each of the materials was subjected to sieving and only the fraction with particle sizes ranging between 212 μm and 300 μm was collected for testing. Hence, all tested sands have the same coefficient of uniformity ($C_u = 1{\cdot}2$). To quantify particle shape for each material, a laser scanning and imaging based system known as QicPic was used. Compared with the traditional visual inspection method, the new method can provide much more efficient and reliable measurements of particle shape. A description of using the robust system to measure particle shape can be found in Altuhafi *et al.* (2013) and Yang & Luo (2015).

For a given particle, three shape measures can be made using the system, including aspect ratio (AR), convexity (C) and sphericity (S). Based on the binary images generated from the QicPic, an additional shape measure, known as roundness (R), has also been made in this study. Fig. 2 illustrates the definitions for these shape parameters. To gain reasonably accurate measurements of roundness, each selected binary image was zoomed up at the same scale (×400). For each type of sand, around 60 images were analysed by using a computer-aided tool to minimise subjectivity in measurement. In general, the measured values of roundness follow a normal distribution, with the standard deviation being less than 0·06. The measurements are much more consistent compared with the conventional chart-based method, which has been found to have a standard deviation as high as 0·13 (Zheng & Hryciw, 2015). As an example, Fig. 3 shows the cumulative distribution curves for roundness and convexity for the four sands; for each curve it is reasonable to choose the shape value corresponding to the 50% cumulative distribution as a representative value. A detailed summary of the shape and size data for these sands is given in Table 1. Note that the values of AR, C and S in Table 1 are direct outputs from the QicPic analyser and they may show slight discrepancies compared with those by reading off from the distribution curves in Fig. 3. Based on the results in Table 1 and Fig. 3, it is possible to conclude that, among the four sands tested, Ottawa sand is the most rounded, whereas Fujian A sand is the most angular, with the other two sands in between.

In this study, shear wave velocity was directly measured for sand specimens using the piezoelectric bender elements installed in a resonant column device. The apparatus can accommodate a soil specimen 50 mm in diameter and 100 mm high, with an air-filled cell pressure up to 1 MPa and an internal linear variable differential transducer (LVDT) of high resolution. All specimens were prepared by the dry tamping method and were tested under dry conditions. Each specimen was subjected to isotropic compression in stages, typically at 100, 200, 300, 400 and 500 kPa, and the void ratio

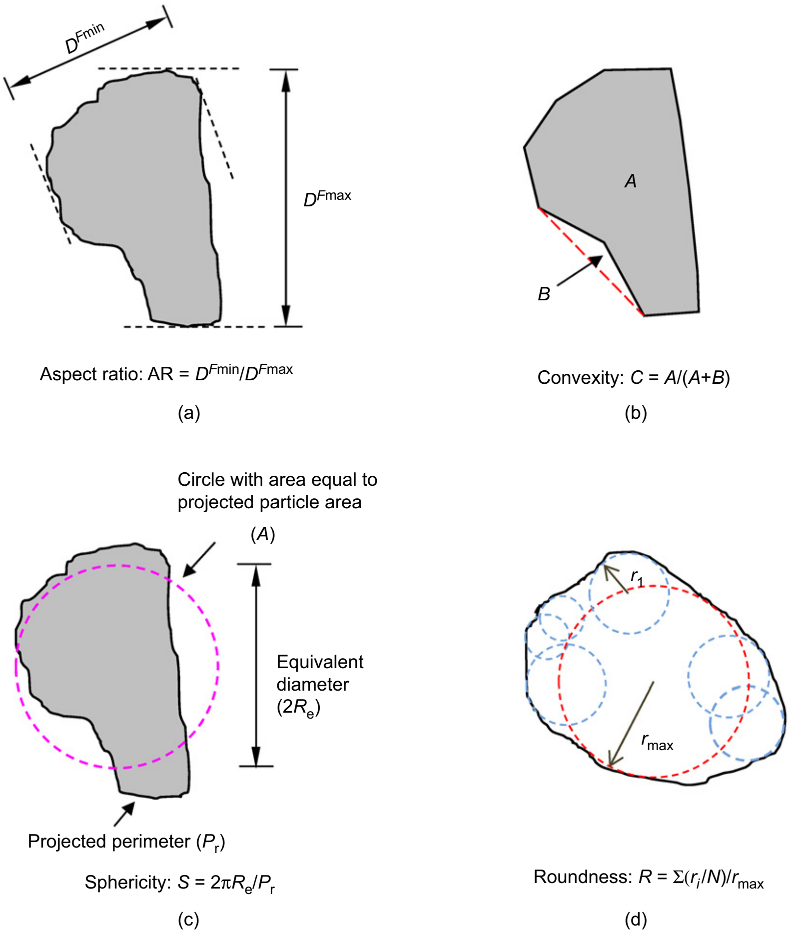

Fig. 2. Schematic illustration of definitions for shape parameters: (a) aspect ratio; (b) convexity; (c) sphericity; (d) roundness

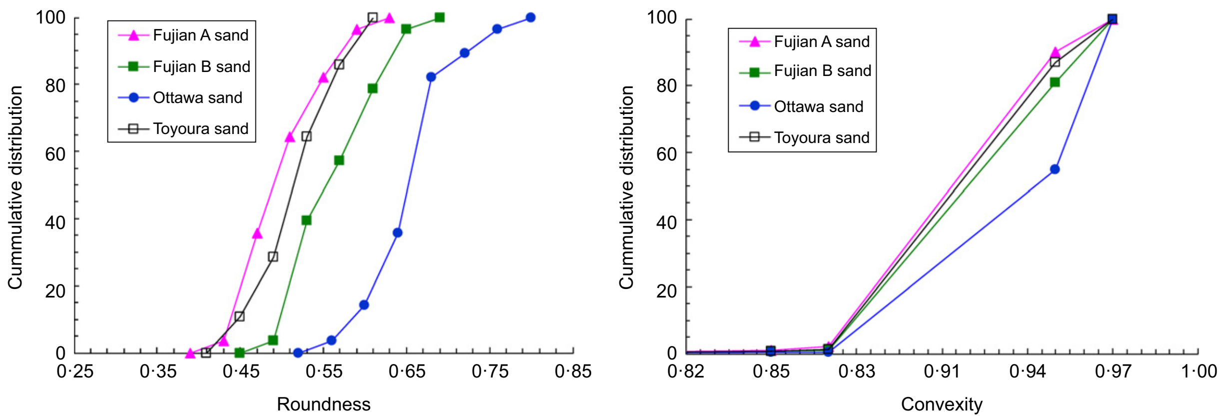

Fig. 3. Cumulative distribution curves of shape parameters of tested materials: (a) roundness; (b) convexity

Table 1. Shape and size data of tested materials

Material	Coefficient of uniformity (C_u)	Mean size, D_{50}: μm	Aspect ratio (AR)	Convexity (C)	Sphericity (S)	Roundness (R)
Ottawa sand	1·2	256	0·762	0·948	0·905	0·652
Toyoura sand	1·2	256	0·750	0·937	0·890	0·513
Fujian A sand	1·2	256	0·738	0·933	0·878	0·499
Fujian B sand	1·2	256	0·742	0·940	0·889	0·560

was updated at each stage. For details about the method for signal interpretation in bender element tests and reliability of results as compared with resonant column tests, the reader may refer to Yang & Gu (2013).

RESULTS AND DISCUSSION

The measured V_s values for the four sands are shown in the four plots in Fig. 4. In each plot, V_s is shown as a function of void ratio at various confining stresses. The effects of

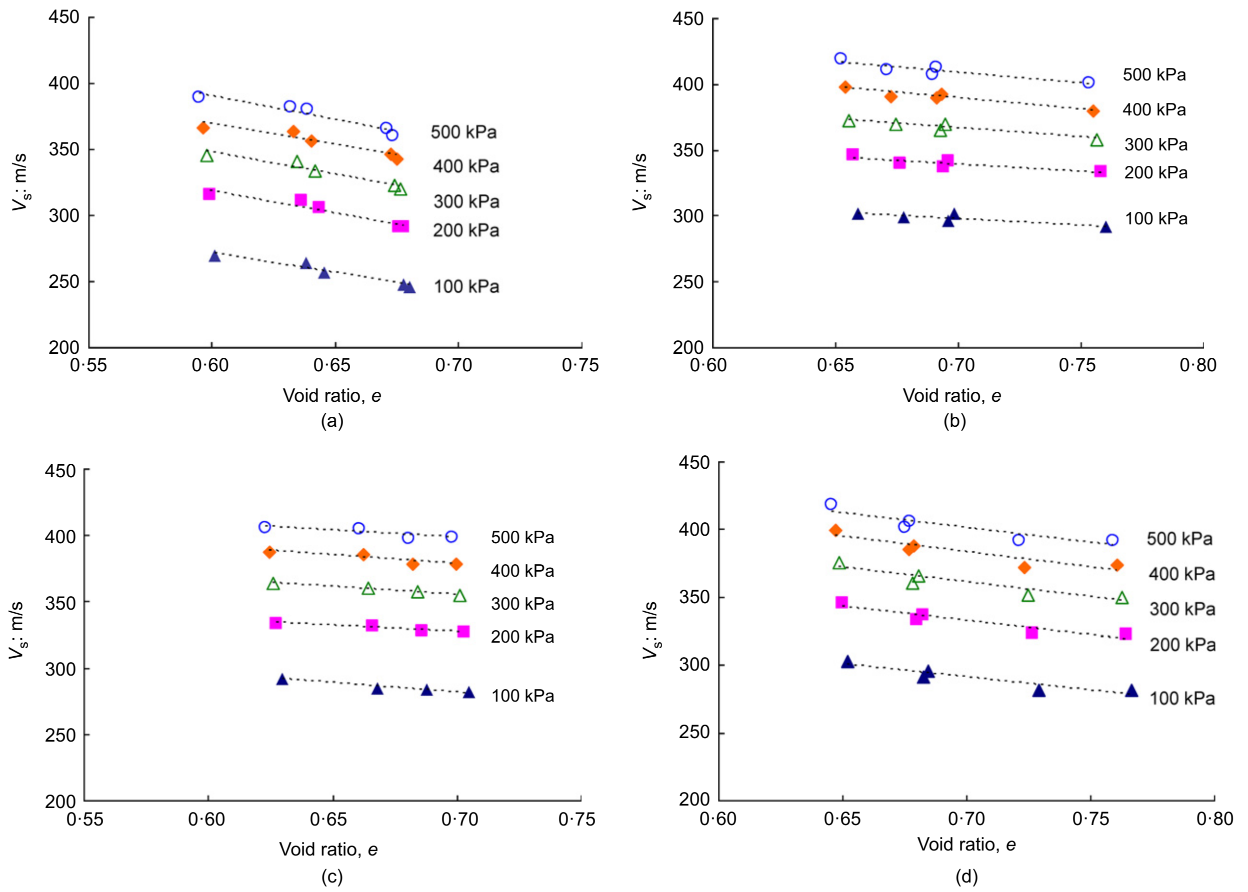

Fig. 4. Measured shear wave velocity as a function of void ratio: (a) Ottawa sand; (b) Fujian A sand; (c) Fujian B sand; (d) Toyoura sand

these two factors on V_s are evident: at a given void ratio, V_s increases with increasing confining stress, whereas at a given confining stress, V_s decreases with increasing void ratio. A direct comparison of any two plots in Fig. 4 suggests that the influence of grain shape exists; for example, for a given confining stress, Fujian sand A tends to have higher V_s values compared with Ottawa sand.

Given that V_s is affected by void ratio and confining stress, a rational examination of the effect of grain shape should be based on comparison that rules out the influence of these two factors; otherwise the observed difference may not be a true reflection of the influence of shape (Bui & Priest, 2007). In doing so, the small-strain stiffness G_0 associated with V_s is determined herein as

$$G_0 = \rho V_s^2 = AF(e)(\sigma'/p_a)^n \quad (1)$$

where ρ is mass density; σ' is confining pressure; p_a is a reference pressure, usually taken as the atmospheric pressure; A and n are two material constants; and $F(e)$ is a function of void ratio (e), typically taking the following form (Iwasaki & Tatsuoka, 1977)

$$F(e) = (2{\cdot}17 - e)^2/(1 + e) \quad (2)$$

In Fig. 5, the void ratio-corrected G_0 (i.e. $G_0/F(e)$) is plotted as a function of normalised confining pressure for Ottawa sand and Fujian A sand, along with the best-fit lines. Clearly, for the entire range of confining pressures, the void ratio-corrected G_0 for Ottawa sand is markedly lower than that for Fujian A sand. Noting that the two quartz sands share the same grading and size, the observed difference is thus mainly attributable to the difference in grain shapes of these two sands. Compared with Fujian A sand, Ottawa sand is much more rounded (see Table 1 and Fig. 3).

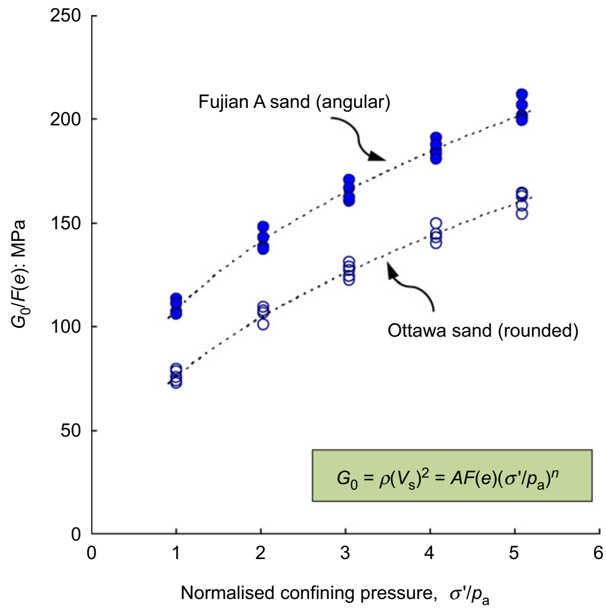

Fig. 5. Void ratio corrected shear modulus as a function of confining pressure for two quartz sands with distinctly different grain shapes

Here, one may be concerned about the potential effect of surface roughness on shear wave velocity. Santamarina & Cascante (1998) studied this effect using resonant column tests on steel balls. To make distinctly different surface roughness, the steel balls were either washed using Alconox detergent (referred to as the mild rust specimen) or washed using Alconox detergent and then submerged in a

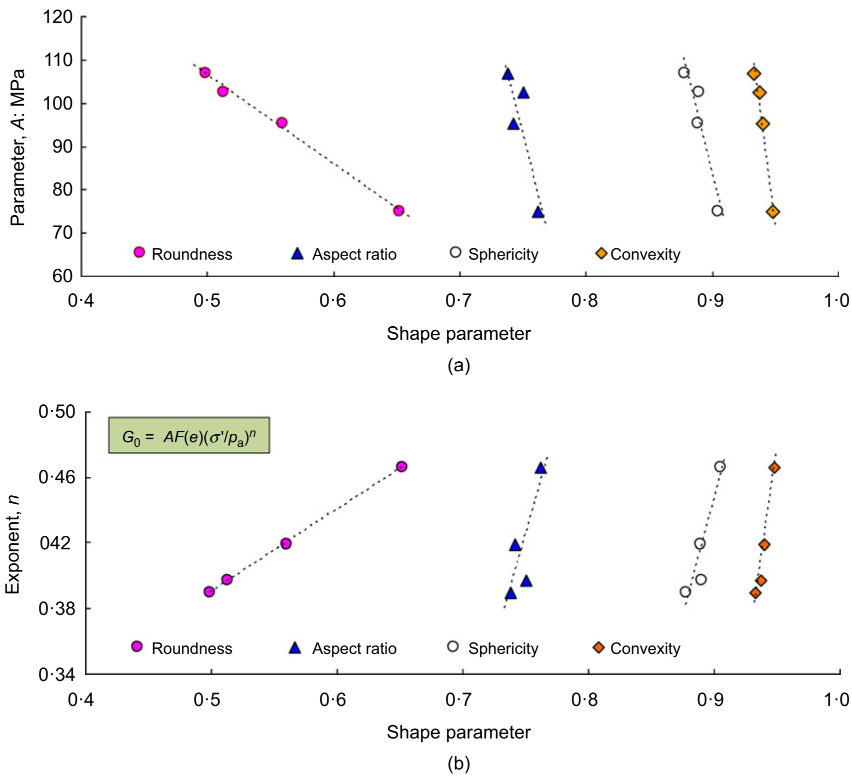

Fig. 6. Dependence of stiffness parameters on grain shape: (a) parameter *A*; (b) exponent *n*

hydrochloric acid solution (referred to as the rusted specimen). Compared with the mild rust specimen, the rusted specimen showed lower shear wave velocity. Although it is an interesting study, it is worth noting that rusting not only alters the surface roughness, but also changes the material chemically. The implication is that the observed change in stiffness may partly reflect the change in the substance's composition. In the present study, the four types of sands all are quarzitic and no deliberate treatment was made to sand grains to alter their surface roughness. According to the studies of Alshibli & Alsaleh (2004) and Otsubo *et al.* (2015), the variation of surface roughness for natural quartz sands is quite small as compared with that of soil analogues (e.g. glass beads or steel balls) after deliberate treatment. Altuhafi *et al.* (2016) made an effort to examine the effect of roughness on small-strain stiffness of natural sands, but did not find any clear effect. In this context, the effect due to a small difference in roughness is considered insignificant as compared with the effect of shape in the present study.

To further quantify the effect of grain shape, the two parameters in equation (1), A and n, were determined for each of the four sands, and then were plotted as a function of the four shape parameters, as shown in Fig. 6. In general, a reasonable correlation exists between either A or n and any one of the shape parameters. The parameter A tends to decrease with increasing shape parameter, whereas the exponent n shows an opposite trend, in that it increases with increasing shape parameters. Values of the stress exponent fall in a narrow range, varying from 0·39 to 0·47, which are typical of uniform quartz sands reported in the literature. While a reasonable correlation is observed, there is a concern that, except for roundness, the range of the other three shape parameters (AR, C and S) is limited. The narrow range of the three parameters as compared with roundness has also been observed on a wide range of natural sands (Altuhafi *et al.*, 2016), implying that it is likely to be an inherent characteristic of natural sand grains. To further explore the

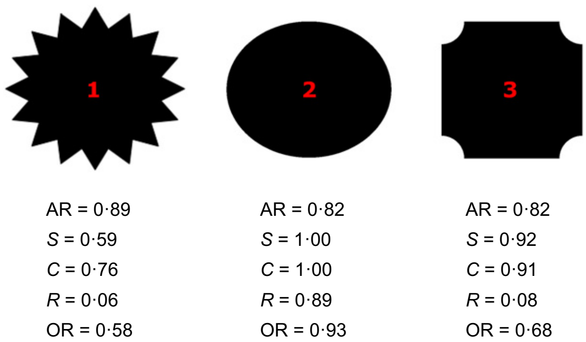

Fig. 7. Shape parameters determined for three idealised irregular particles

reason behind the observation, an attempt is made herein by examining three irregular geometries that are of similar areas, but with distinctly different shapes, as shown in Fig. 7. For each geometrical shape, the four shape parameters are determined in accordance with the methods given in Fig. 2 and the determined values are included in Fig. 7 for ease of reference. It is interesting to note that roundness is much more sensitive to sharp corners than the other three shape parameters, thus yielding a much broader range (0·06–0·89) as compared with aspect ratio, sphericity and convexity.

To take into account these shape parameters in a collective manner, an additional shape index, referred to as the overall regularity (OR), is introduced as follows

$$\mathrm{OR} = (\mathrm{AR} + S + C + R)/4 \tag{3}$$

This overall regularity is an improved version of that defined by Yang & Luo (2015) in which roundness was not included in making the average. It also differs from the regularity defined by Cho *et al.* (2006), which was taken as the average of sphericity and roundness only. In this respect,

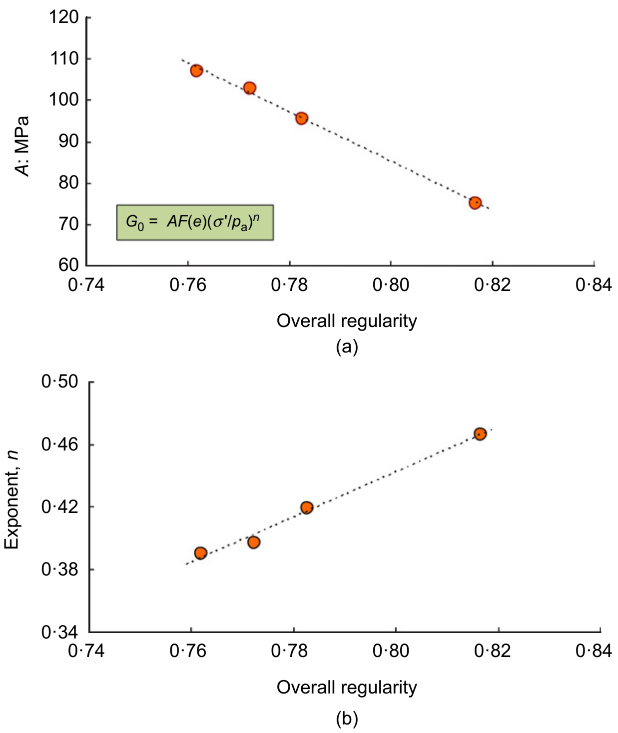

Fig. 8. Correlation of stiffness parameters with overall regularity of sand grains: (a) parameter *A*; (b) exponent *n*

the overall regularity in equation (3) may serve as a more comprehensive shape measure, although it is a non-physical quantity.

Now, the two stiffness parameters (*A* and *n*) are plotted as a function of overall regularity using the test data, as shown in Fig. 8. A reasonable correlation is evident, showing that parameter *A* decreases with increasing OR while the exponent *n* increases with increasing OR. The combined effect is that V_s or G_0 will decrease as sand grains become more rounded and regular. From the perspective of micro-mechanics (Gu & Yang, 2013; Yang & Liu, 2016), this result is considered reasonable – that is, under otherwise similar conditions in terms of void ratio, confining pressure, grading and minerology, a sand specimen with angular grains tends to have a higher coordination number (i.e. number of contacts per particle) than a sand specimen with rounded grains and, as a consequence, tends to show a higher G_0 or V_s.

CONCLUSIONS

The intention in this technical note is to provide a quick communication about the fundamental question as to whether the shear wave velocity (V_s) of sand is dependent on particle shape. The main points are summarised as follows.

(*a*) The specifically designed experiments offer solid evidence that V_s or the associated small-strain stiffness G_0 is dependent on grain shape. Under otherwise similar conditions, a sand specimen with angular particles tends to exhibit higher V_s or G_0 values than a sand specimen with rounded particles.

(*b*) The effect of grain shape can be characterised through two stiffness parameters (*A* and *n* in equation (3)), which reasonably correlate with shape parameters. Using the OR as a shape measure collectively accounting for particle roundness, sphericity, aspect ratio and convexity, the stiffness parameter *A* was found to decrease with increasing OR, whereas the exponent *n* shows an opposite trend.

(*c*) When studying the shape effect on sand stiffness and on other mechanical properties, it is important to rule out the effects of particle size, grading and mineralogy, and also it is important to properly account for the influence of void ratio and confining pressure. Otherwise, misleading conclusions may be reached.

(*d*) Because the laboratory experiments were conducted under well-controlled conditions, the data presented herein can serve as a useful reference for validation and calibration of numerical and analytical developments in the area. Further work to validate the findings and to refine the correlations using new experimental data would be worthwhile.

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NOTATION

A	coefficient in equation (1)
C	convexity
C_u	coefficient of uniformity
D_{50}	mean particle size
$D^{F\max}$, $D^{F\min}$	maximum and minimum Feret's diameter, respectively (Fig. 2)
e	void ratio after consolidation
$F(e)$	void ratio function
G_0	small strain shear stiffness
n	stress exponent
P_r	projected perimeter
p_a	reference stress
R	roundness
r_i	radius of a circle fitted to the *i*th particle corner (Fig. 2)
r_{max}	radius of the maximum inscribed circle (Fig. 2)
S	sphericity
V_s	shear wave velocity
α, β	parameters to characterise V_s in Fig. 1
ρ	mass density
σ'	effective confining stress

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