Discrete Element Analysis of the K_0 of Granular Soil and Its Relation to Small Strain Shear Stiffness

Xiaoqiang Gu[1]; Jing Hu[2]; Maosong Huang[3]; and Jun Yang, F.ASCE[4]

Abstract: The discrete element method (DEM) was used to investigate the coefficient of earth pressure at rest, K_0, of granular soils. The results indicate that K_0 decreases as the void ratio decreases and the vertical stress increases. The K_0 of specimens prepared by different methods may be quite different despite the void ratio and vertical stress being the same. The analysis reveals that at a particulate level, the coordination number of the soil determines the K_0. Both DEM simulation and laboratory data indicate that there is a good relationship between the K_0 and the small strain shear stiffness. **DOI: 10.1061/(ASCE)GM.1943-5622.0001102.** *© 2018 American Society of Civil Engineers.*

Author keywords: Coefficient of earth pressure at rest; Granular soil; Discrete element method; Small strain shear stiffness.

Introduction

The coefficient of the earth pressure at rest, K_0, plays an essential role in determining the geostatic stress state, which significantly affects soil behavior and thus, the analysis and design of geotechnical structures. In the past, tremendous studies have been carried out to investigate the factors influencing the K_0 and the prediction of it (Jaky 1948; Mayne and Kulhawy 1982; Okochi and Tatsuoka 1984; Chu and Gan 2004; Guo 2010; Gao and Wang 2014). For example, Okochi and Tatsuoka (1984) systematically investigated the factors that affect the K_0 value of Toyoura sand in a triaxial apparatus, including the void ratio, sample preparation method, and stress history. Mayne and Kulhawy (1982) analyzed the K_0 values of over 170 soil specimens in previous works and proposed an equation to predict the K_0 value to account for the stress history. In the laboratory, the K_0 can generally be measured by an oedometer with lateral stress measurement or by a triaxial apparatus. It was found that the K_0 is affected by many factors, such as void ratio (Okochi and Tatsuoka 1984; Guo 2010), friction angle (Jaky 1948; Mayne and Kulhawy 1982), stress history (Mayne and Kulhawy 1982; Okochi and Tatsuoka 1984), confining pressure (Okochi and Tatsuoka 1984), sample preparation method (Chu and Gan 2004; Northcutt and Wijewickreme 2013), aging (Gao and Wang 2014), particle shape (Andrawes and El-Sohby 1973; Guo 2010), and mineral of the particle (Andrawes and El-Sohby 1973). Of these, void ratio, friction angle, and stress history are commonly recognized as the most important factors.

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Note. This manuscript was submitted on February 1, 2017; approved on September 28, 2017; published online on January 5, 2018. Discussion period open until June 5, 2018; separate discussions must be submitted for individual papers. This technical note is part of the ***International Journal of Geomechanics***, © ASCE, ISSN 1532-3641.

It is generally agreed that the K_0 value of granular soils increases as the void ratio increases, which means loose soils have larger K_0 values [e.g., Andrawes and El-Sohby (1973); Chu and Gan (2004); Feda (1984); Guo (2010); Okochi and Tatsuoka (1984)]. Okochi and Tatsuoka (1984) investigated the K_0 value of Toyoura sand in a modified triaxial apparatus and found that the K_0 value increased as the void ratio increased, as shown in Fig. 1(a). Moreover, even for the same void ratio, the K_0 value of the specimen prepared by air pluviation was much larger than that prepared by wet tamping. Northcutt and Wijewickreme (2013) measured the K_0 value of Fraser river sand using an oedometer in which the lateral stress was measured by strain gauges and reported similar results, as shown in Fig. 1(b). Generally, it is deduced that such a phenomenon is due to the difference of soil microstructure (fabric) resulting from different sample preparation methods. Indeed, for sand specimens prepared by these two methods, Yang et al. (2008) measured distinctly different distributions of particle orientation at the grain level. However, the kind of microstructure that determines the K_0 of granular soil and how the void ratio and sample preparation method affect the microstructure are not yet well understood.

Because of the difficulty in measuring the K_0 directly in the field, Jaky's equation (Jaky 1944, 1948) is widely used to predict the K_0 as $K_0 = 1 - \sin\varphi$, in which φ is the effective internal friction angle. Both the peak internal friction angle, φ_p [e.g., Feda (1984); Wanatowski and Chu (2007); Watabe et al. (2003)] and the critical state internal friction angle, φ_c [e.g., Gao and Wang (2014); Mesri and Hayat (1993); Northcutt and Wijewickreme (2013)] were used in the literature. Nevertheless, agreement on which friction angle would be more appropriate for predicting the K_0 in Jaky's equation has not been reached. Moreover, although Jaky's equation has been sufficiently accepted for engineering practice, the theoretical derivation of it was criticized by Handy (1985) and Michalowski (2005) and the accuracy of it was questioned through consideration of experimental data [e.g., Feda (1984); Chu and Gan (2004); Guo (2010)]. In the authors' view, it is not surprising that the K_0, being a parameter of the current state, cannot be predicted well by the friction angle, a parameter of the failure state (i.e., far removed from current state).

From the viewpoint of micromechanics, the mechanical behavior of granular materials depends on the microstructure of soil, such as contacts through which the soil bears and transfers the loads. Several attempts have been made in the laboratory to measure the microstructure of the granular material during loading (Oda et al.

Int. J. Geomech., 2018, 18(3): 06018003

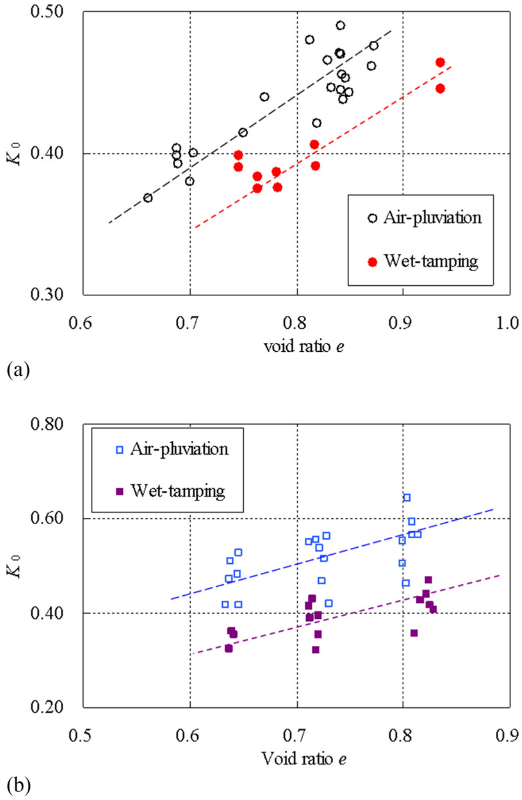

Fig. 1. Effect of void ratio and sample preparation method on the K_0 [(a) data from Okochi and Tatsuoka 1984; (b) data from Northcutt and Wijewickreme 2013]

1985; Majmudar and Behringer 2005; Yang et al. 2008). For example, Majmudar and Behringer (2005) measured the distribution of contact forces of photoelastic disks in isotropic compression and pure shearing. However, it is generally difficult and tedious to obtain the microstructure of the soil in the laboratory and impossible to obtain all the important aspects of the microstructure. It is fortunate that the discrete element method (DEM), proposed by Cundall and Strack (1979), provides a powerful tool to study the macroscopic behavior of soil at a microscopic level. DEM has been extensively used to reveal many macroscopic soil behaviors successfully at the particulate level, such as the shear behavior at large strain (Muir Wood and Maeda 2008; Thornton 2000; Gu et al. 2014; Huang et al. 2014), small strain behavior of granular material (Ng and Petrakis 1996; Gu et al. 2017), the K_0 value of granular material (Gao and Wang 2014; Gu et al. 2015a), and so forth. For example, Gao and Wang (2014) investigated the changes in K_0 value during the primary compression and secondary compression by experiments and DEM simulations. However, they did not reveal the underlying mechanics related to the K_0 value of granular material. Considering the advantages it confers, such that identical specimens can be prepared and all the microstructures of the soil can be can be conveniently obtained, DEM can be used to investigate how the macroscopic factors observed in the laboratory affect the microstructure of the soil and how the microstructure of the soil affects the K_0 value, which constitutes the main aim of this study.

Motivated by the review described in this section, in this study DEM was used to simulate the one-dimensional (1D) compression (oedometer) test to determine the K_0 value of granular soil at different void ratios and confining pressures and to explore the underlying fundamental mechanism. Attempts were made to relate the K_0 value to the small strain shear stiffness, which is also a current state parameter of the soil and depends on the void ratio, confining pressure, and sample preparation method [e.g., Alba et al. (1984); Ezaoui and Di Benedetto (2009); Gu et al. (2015b)]. The result confirms that the K_0 decreases as the void ratio decreases and the vertical stress increases. It reveals that at a particulate level, the mechanical coordination number (i.e., average number of contacts per particle) of the granular soil determines the K_0 and it probably explains the effect of the sample preparation method on the K_0. It was also found that the K_0 value of a certain soil may be evaluated by the small strain shear stiffness, which can be easily determined by the shear wave measurement.

DEM Modeling

The numerical specimen was represented by a cubic space of 1,000 cm^3 confined by three pairs of frictionless walls, and the stiffness of walls was 10^{10} N/m. The particles were represented by spheres, and the particle size distribution of Toyoura sand was used. The nonlinear Hertz-Mindlin contact law was adopted. As suggested by Gao and Wang (2014) and Ng and Petrakis (1996), the shear modulus and Poisson's ratio of the particle were chosen as 5.8 GPa and 0.15, respectively.

First, spheres were randomly generated in the cubic space and deposited under gravity force after particle generation. After gravity deposition, the specimen was compressed one dimensionally in the vertical direction to reach a vertical stress of 1 kPa. It is noteworthy that, during the deposition, different interparticle friction coefficients, designated by μ, were used temporally to generate specimens with different densities (Muir Wood and Maeda 2008; Yan and Dong 2011; Yang and Dai 2011). Generally, low μ values generate dense specimens because slippage at the contacts and the rearrangement of the particle easily occur, while high μ values generate loose specimens. For this study, μ values of 0.001, 0.05, and 0.4 were used to generate three specimens with a void ratio, e, at a vertical stress of 1 kPa, of 0.591, 0.637, and 0.667, respectively. It is worth emphasizing that the μ values used in the deposition process do not represent the real friction coefficients of sands. Therefore, to obtain the more realistic behavior, μ was changed to the measured value of 0.5 in quartz (Dobry and Ng 1992), and these specimens were denoted as A_e = 0.591, B_e = 0.637, and C_e = 0.667. Moreover, to prepare specimens with similar void ratios by different sample preparation methods, a certain amount of particles in the specimen A_e = 0.591 was randomly removed to obtain two other specimens with an e of 0.640 and 0.668 and denoted as A′_e = 0.640 and A′_e = 0.668, respectively. Similarly, a specimen with e of 0.668 was also prepared by removing particles from the specimen B_e = 0.637, and it was denoted as B′_e = 0.668. Finally, numerical 1D compression tests in the vertical direction (i.e., oedometer test) were performed on the six generated specimens to evaluate the K_0 value. During the compression, the evolution of the soil microstructure was monitored. Furthermore, numerical drained triaxial probe tests (Gu and Yang 2013; Gu et al. 2013) were also carried out to determine the small strain shear modulus of the specimen at each K_0 stress state. The schematic diagram of the detailed DEM simulation is shown in Fig. 2.

Results and Discussions

Fig. 3 shows the evolutions of K_0 in the oedometer tests for specimens with different initial void ratios. It is interesting to note that the K_0 remarkably increased as the initial void ratio increased. This phenomenon is consistent with the experiment results by Andrawes

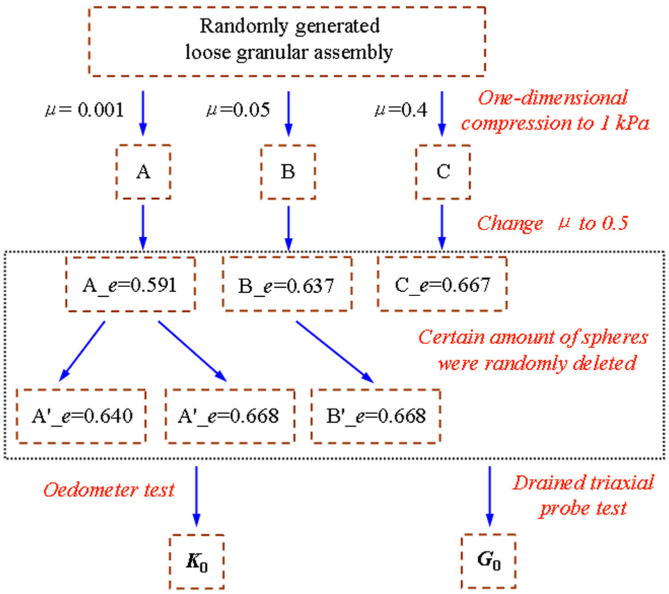

Fig. 2. Schematic diagram of the DEM simulation

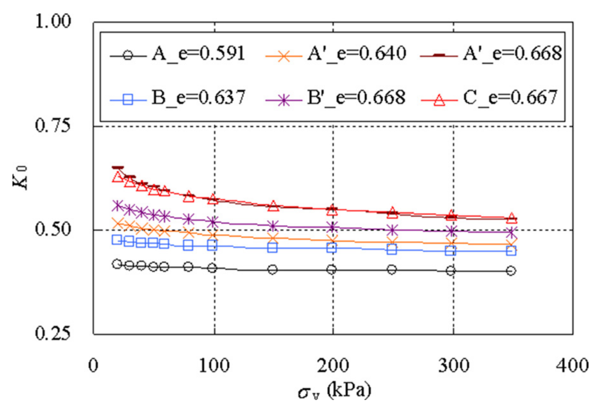

Fig. 3. Effects of confining pressure and void ratio on the K_0

and El-Sohby (1973) and Guo (2010). Meanwhile, the K_0 value decreased as the vertical stress increased, especially at the beginning of the loading and for the loose specimen. For a better illustration, the K_0 was plotted against the void ratio at each stress state and the plot is presented in Fig. 4. It is clear that the initial void ratio has a significant effect on the K_0 and that such an effect must be accounted for in the K_0 prediction. It is noteworthy that the void ratio shows a much more significant effect on the K_0 than the increase of vertical stress does. Even at the same void ratio, the K_0 value may be quite different for different specimens (e.g., C_e = 0.667, A′_e = 0.668, and B′_e = 0.668). This phenomenon is consistent with the observation in the laboratory that at the same void ratio, the K_0 values of specimens prepared by different sample preparation methods are quite different (Fig. 1). These findings suggest that the void ratio is not an intrinsic mechanical parameter determining the K_0, but it has an important effect on the intrinsic mechanical parameter that determines the K_0.

Keeping in mind that the granular materials bear and transfer the load via contacts, it is expected that the number of contacts in a soil plays an important role in the K_0 behavior (Gu et al. 2015a). Fig. 5 shows the relationship between K_0 and the mechanical coordination number, CN. The mechanical coordination number describes the average number of contacts per particle, representing the contact

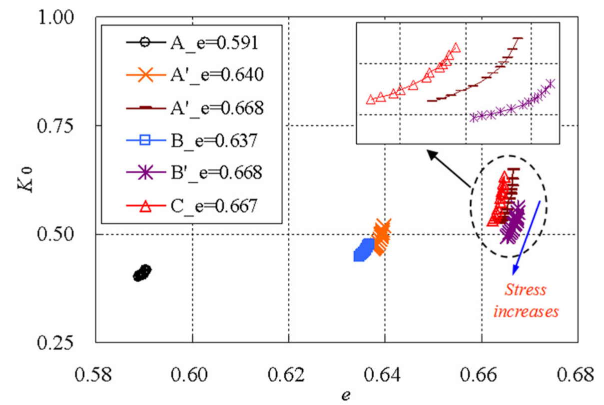

Fig. 4. Relationship between the K_0 and void ratio

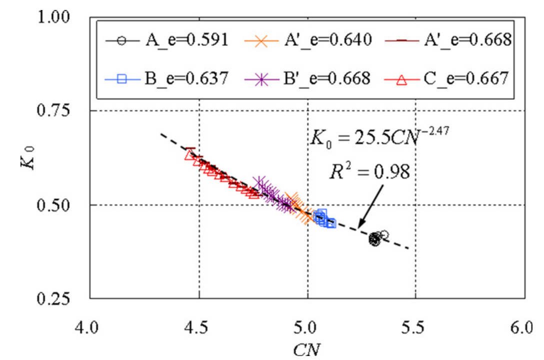

Fig. 5. Relationship between K_0 and coordination number CN

density in the soil. It is noteworthy that only the particles with more than two contacts were taken into account to calculate the total number of contacts, while all the particles were used to evaluate the void ratio (Thornton 2000). It is interesting to note that the K_0 continuously decreased as CN increased. The evolution of CN successfully explains the decrease of K_0 with increasing vertical stress and the difference of K_0 between specimens with nearly the same void ratios but that were prepared by different methods. The findings indicate that with higher contact density, the specimen can resist the axial load more effectively and induce a smaller lateral stress increment, thus resulting in a smaller K_0 value. The macroscopic decreasing of K_0 with decreasing void ratio and increasing vertical stress as well as the effect of sample preparation methods can be uniquely reflected by the evolution of the coordination number.

Fig. 6 shows the comparison between the K_0 values from the DEM and those predicted by Jaky's equation. Numerical drained triaxial tests were performed on the specimens to obtain the friction angle. The specimens with initial void ratios of 0.591 and 0.637 showed strain-softening behaviors, and the peak friction angle for each, φ_p, was 26.6 and 23.5°, respectively. The specimen with an initial void ratio of 0.667 showed a hardening behavior without a peak friction angle. The critical state friction angle, φ_c, for each of the three specimens was 21.0°, which is consistent with critical state soil mechanics. As seen in Fig. 6, it is clear that the use of φ_p provided a better prediction of K_0 than the use of φ_c; however, the prediction was still remarkably higher than the DEM result. Theoretically, the use of φ_p might reflect the effect of the void ratio on the K_0 value because the void ratio affects the φ_p. The φ_p increased as the void ratio decreased, which indicates that Jaky's equation successfully predicts the decrease in K_0 value as the void

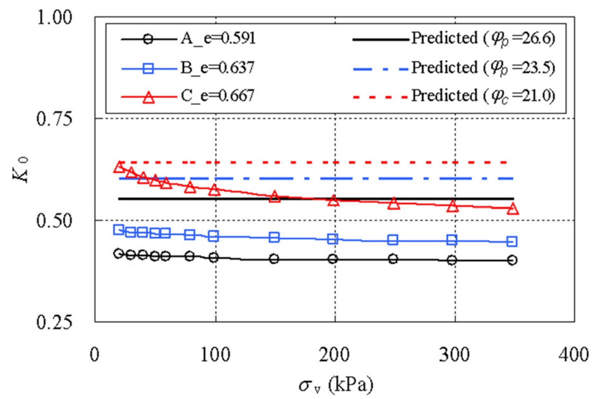

Fig. 6. Comparison of K_0 obtained by DEM simulation and predicted by Jaky's equation

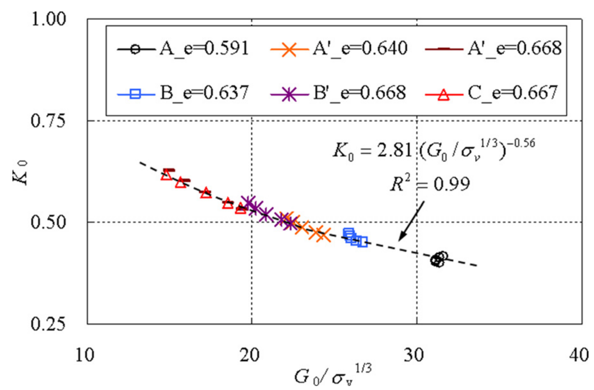

Fig. 7. Relationship between K_0 and normalized small strain shear modulus in the DEM (G_0 in megapascals and σ_v in kilopascals)

ratio decreases. However, φ_c will be the same for specimens with different initial void ratios, and therefore, using φ_c in Jaky's equation cannot reflect the observed decrease in K_0 with a decreasing void ratio. Moreover, it is not surprising that the K_0, a parameter at the current state, cannot be successfully predicted by the friction angle, a parameter at the failure state (i.e., far removed from the current state).

Like the K_0, the small strain shear stiffness, G_0, is a parameter at the current state, and it can be conveniently determined by the shear wave measurement using bender elements (BE). Moreover, it also depends on the void ratio, confining pressure, sample preparation method (Alba et al. 1984; Ezaoui and Di Benedetto 2009; Gu et al. 2015b). Therefore, attempts were made to relate the K_0 to the G_0. Gu and Yang (2013) showed that the small strain shear stiffness is proportional to the mechanical coordination number and the stress to a power of 1/3 according to the Hertz-Mindlin contact law. Meanwhile, the results of this study showed that the K_0 value depends only on the mechanical coordination number (Fig. 5), but it is generally independent of the stress level (Fig. 3). Therefore, the K_0 was plotted against the small strain shear modulus normalized by the vertical stress, $G_0/\sigma_v^{1/3}$, as shown in Fig. 7. The good relationship between the K_0 and $G_0/\sigma_v^{1/3}$ is interesting. To verify such a relationship, Fig. 8 shows the plots the K_0 of Toyoura sand specimens prepared by air pluviation and wet tamping from Okochi and Tatsuoka (1984) against $G_0/\sigma_v^{1/3}$. The G_0 (in megapascals) of the Toyoura sand specimens prepared by air pluviation and wet tamping were calculated by $G_0 = 86.5\left[(2.17 - e)^2/1 + e\right](\sigma_0'/p_a)^{0.45}$ and $G_0 = 104.6\left[(2.17 - e)^2/1 + e\right](\sigma_0'/p_a)^{0.40}$, according to the

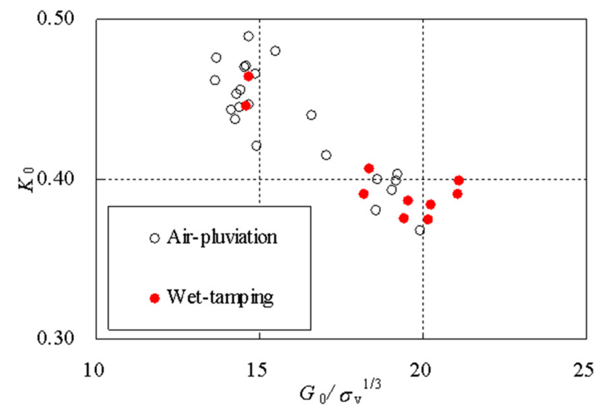

Fig. 8. Relationship between the K_0 and normalized small strain shear modulus for Toyoura sand in the laboratory (G_0 in megapascals and σ_v in kilopascals)

bender element measurements by Gu et al. (2015b). In this case, e is the void ratio, σ_0' is the mean effective stress (in kilopascals), and p_a is a reference stress of 98 kPa. As seen in Fig. 8, the relationship between the K_0 and $G_0/\sigma_v^{1/3}$ becomes unique in spite of different void ratios and sample preparation methods. Therefore, the small strain shear stiffness may be used to evaluate the K_0. Obviously, further research is needed to verify this result.

Summary and Conclusions

In this study, the K_0 values of normally consolidated granular materials were investigated by DEM simulations. Attempts were made to explore the fundamental mechanism related to the K_0 and link the K_0 with the small strain shear stiffness of the soil. The main findings of this study can be summarized as follows:

1. The results showed that the K_0 decreased as the void ratio decreased and the vertical stress increased during the numerical oedometer tests, and it may be quite different for specimens prepared by different methods despite having the same void ratios and vertical stresses.
2. The results revealed that the K_0 of a certain soil depends on the mechanical coordination number, which describes the average number of contacts per particle at the microscopic level. The observed effects of the confining pressure, void ratio, and sample preparation method on the K_0 in the laboratory can be explained by the difference in mechanical coordination numbers of the tested specimens.
3. The DEM simulation showed that there is a good relationship between the K_0 and the small strain shear stiffness, which can be determined by shear wave measurement. The preliminary analysis of the laboratory test data indicates that the small strain shear stiffness may provide an effective way to evaluate the K_0 value in practice.

Acknowledgments

The work presented in this paper was supported by the National Key Research and Development Program (Grant 2016YFC0800204), National Natural Science Foundation of China (Grants 51308408 and 51428901), and State Key Laboratory for Geomechanics and Deep Underground Engineering, China University of Mining and Technology (CUMT) (Grant SKLGDUEK1712). These supports are gratefully acknowledged.

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