

# Saturation effects on horizontal and vertical motions in a layered soil–bedrock system due to inclined SV waves

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## Abstract

A study of the effects of pore-water saturation on the horizontal and vertical components of ground motions in a multi-layered soil–bedrock system due to inclined SV waves is presented. Both the soil and the rock are modeled as a partially water-saturated porous medium which is characterized by its degree of saturation, porosity, permeability, and compressibility. An efficient formulation is developed for the computation of the two-dimensional ground motions, which are considered as functions of the angle of incidence, the degree of saturation, the frequency, and the geometry of the system. Numerical results for both the half-space model and the single-layered model indicate that the effect of saturation may be significant, and is dependent on the angle of the incidence. Even a slight decrease of full saturation of the overlying soil may cause appreciable difference in the amplitudes of ground motions in both the horizontal and vertical components and the amplitude ratios between the two components at the ground surface, implying that one may need to carefully take into account the saturation conditions in the interpretation of field observations. © 2001 Elsevier Science Ltd. All rights reserved.

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## 1. Introduction

It has long been recognized that, because of groundwater, sedimentary soils and rocks may be appropriately represented as a water-saturated porous medium which is composed of a solid skeleton and water-filled pore spaces. In reality, however, partial saturation conditions may occur in certain situations due to fluctuating water tables, flooding, and recharge of groundwater. Partial saturation conditions may exist in offshore sites constructed by land reclamation [1], or in marine sediments as evidenced by experiments [2]. Recent analyses of the downhole array observations during the Hyogo-ken Nanbu (Kobe) earthquake of 1995 have revealed that pore-water saturation of shallow soils may have a significant affect on the amplification of vertical ground motion [3], raising a great interest in the importance of saturation conditions in interpreting field observations. It would be extremely valuable to investigate the influence of saturation on ground motions in both horizontal and vertical components at a porous soil/rock interface. Such a study may also make potential contributions to the site-evaluation technique generally known as H/V, which is based on the

interpretation of field observations on both the horizontal and vertical components of microtremors/ground motions and has increasingly drawn attention in engineering practice [4,5].

For this reason, a research has been initiated to investigate the effects of saturation on two-dimensional ground motions at a porous soil interface. The problem of a P or SV wave incident at the interface between the rock formation and overlying half-space soil media has recently been studied [6]. In the analysis the soil was modeled as a partially saturated medium with a small amount of air inclusions while the rock was regarded as an ordinary solid. The results indicated that the degree of saturation indeed may cause a significant influence on the motion amplitudes in both the horizontal and vertical components at the interface. Following that fundamental analysis, in this study a problem of more interest in practice is considered, which corresponds to an inclined SV wave incident from the underlying rock formation at the base of a multi-layered soil system with a free surface. This model has been appreciated to be of importance and has been extensively discussed by assuming the soil and rock as an ordinary solid [7,8]. The methods of these analyses are essentially based on the well-known Thomson–Haskell’s matrix formulation [9]. To account for the effect of pore-water saturation, in the present analysis both the soil and the rock are more

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appropriately modeled as a partially water-saturated porous medium that is characterized by its degree of saturation, porosity, permeability, and compressibility. An efficient formulation for the computation of the two-dimensional ground motions is successfully developed. Numerical results are provided to illustrate the saturation effects on the displacement amplitudes in both horizontal and vertical components and the ratios between them.

## 2. Theoretical formulation

### 2.1. Governing equations and general solutions

For a partially water-saturated porous medium, the pore fluid is a mixture of water and air. The relative proportions of constituent volumes can be characterized by its porosity  $n$  and the degree of saturation  $S_r$  as

$$n = \frac{V_v}{V_t} \quad (1a)$$

$$S_r = \frac{V_w}{V_v} \quad (1b)$$

in which  $V_v$  and  $V_w$  are the volumes of pores and pore water, respectively, and  $V_t$  is total volume. A typical case of partial saturation is when the degree of saturation is sufficiently high (e.g. higher than 90%) so that the air is embedded in pore water as bubbles. For this case the mixture can be approximately regarded as a homogeneous pore fluid whose compressibility is dependent on the compressibility of pore water, degree of saturation, and the absolute pore pressure as [10]

$$K_f = \frac{1}{\frac{1}{K_w} + \frac{1 - S_r}{p_a}} \quad (2)$$

in which  $K_f$  is the bulk modulus of pore fluid,  $K_w$  the bulk modulus of pore water, and  $p_a$  is absolute fluid pressure. It can be readily shown that even a very small amount of air in soil can drastically reduce the bulk modulus of pore fluid.

Introducing the concept of homogeneous pore fluid into the theory of two-phase porous media [11], the governing equations can be given in terms of displacements as

$$\mu \nabla^2 \mathbf{u} + (\lambda + \alpha^2 M + \mu) \nabla e - \alpha M \nabla \zeta = \rho \ddot{\mathbf{u}} + \rho_f \ddot{\mathbf{w}} \quad (3a)$$

$$\alpha M \nabla e - M \nabla \zeta = \rho_f \ddot{\mathbf{u}} + \frac{\rho_f}{n} \ddot{\mathbf{w}} + \frac{\eta}{k} \dot{\mathbf{w}} \quad (3b)$$

where  $\mathbf{u}$  and  $\mathbf{w}$  are, respectively, the displacement vectors of solid skeleton and pore fluid with respect to solid phase;  $e = \text{div} \mathbf{u}$  and  $\zeta = -\text{div} \mathbf{w}$  denote, respectively, the volumetric strain of the solid skeleton and the increment of fluid content;  $\eta$  is fluid viscosity and  $k$  is permeability (with the unit  $\text{m}^2$ );  $\rho$  is total density and  $\rho_f$  is the density of pore fluid;  $\lambda$  and  $\mu$  are Lamé constants of solid skeleton;  $\alpha$  and  $M$  are parameters accounting for the compressibility

of grains and fluid, they can be given as

$$\alpha = 1 - \frac{K_b}{K_s} \quad M = \frac{K_s^2}{K_d - K_b} \quad (4)$$

$$K_d = K_s \left[ 1 + n \left( \frac{K_s}{K_f} - 1 \right) \right]$$

where  $K_s$  and  $K_b$  are bulk moduli of solid grains and skeleton, respectively;  $K_f$  is the bulk modulus of pore fluid, it is related to the bulk modulus of pore water, absolute fluid pressure and degree of saturation as shown in Eq. (2).

It should be noted that  $k$  in the formulation is different from the permeability coefficient  $k'$  (m/sec) that is used in soil mechanics and they are related by

$$k = k' \frac{\eta}{\rho_f g} \quad (5)$$

in which  $g$  is the gravitation acceleration at which the permeability is measured.

The relationships between stress, pore pressure, and strain entering into the governing equations are as follows:

$$\sigma_{ij} = \lambda e \delta_{ij} + 2\mu \epsilon_{ij} - \alpha \delta_{ij} p_f \quad (6a)$$

$$p_f = M \xi - \alpha M e \quad (6b)$$

$$\epsilon_{ij} = \frac{1}{2} (u_{i,j} + u_{j,i}) \quad (6c)$$

in which  $\sigma_{ij}$  is total stress tensor,  $\epsilon_{ij}$  is strain tensor,  $p_f$  is pore pressure, and  $\delta_{ij}$  is Kronecker delta.

With the aid of the Helmholtz resolution of displacement fields as follows:

$$\mathbf{u} = \text{grad} \varphi_s + \text{curl} \psi_s \quad (7a)$$

$$\mathbf{w} = \text{grad} \varphi_f + \text{curl} \psi_f \quad (7b)$$

where potentials  $\varphi_s$  and  $\psi_s$  are associated with the solid phase of the bulk material, while  $\varphi_f$  and  $\psi_f$  are associated with the flow of pore fluid, Eqs. (3a) and (3b) become

$$\begin{bmatrix} \rho & \rho_f \\ \rho_f & \rho_f/n \end{bmatrix} \begin{Bmatrix} \ddot{\varphi}_s \\ \ddot{\varphi}_f \end{Bmatrix} + \begin{bmatrix} 0 & 0 \\ 0 & \eta/k \end{bmatrix} \begin{Bmatrix} \dot{\varphi}_s \\ \dot{\varphi}_f \end{Bmatrix} - \begin{bmatrix} \lambda + 2\mu + \alpha^2 M & \alpha M \\ \alpha M & M \end{bmatrix} \begin{bmatrix} \nabla^2 & 0 \\ 0 & \nabla^2 \end{bmatrix} \begin{Bmatrix} \varphi_s \\ \varphi_f \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \end{Bmatrix} \quad (8)$$

$$\begin{bmatrix} \rho & \rho_f \\ \rho_f & \rho_f/n \end{bmatrix} \begin{Bmatrix} \ddot{\psi}_s \\ \ddot{\psi}_f \end{Bmatrix} + \begin{bmatrix} 0 & 0 \\ 0 & \eta/k \end{bmatrix} \begin{Bmatrix} \dot{\psi}_s \\ \dot{\psi}_f \end{Bmatrix} - \begin{bmatrix} \mu & 0 \\ 0 & \mu \end{bmatrix} \begin{bmatrix} \nabla^2 & 0 \\ 0 & \nabla^2 \end{bmatrix} \begin{Bmatrix} \psi_s \\ \psi_f \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \end{Bmatrix} \quad (9)$$

Eqs. (8) and (9) describe the propagation of compressional and shear waves in the porous media, respectively. For harmonic plan waves, two eigenvalue equations may be derived from these equations [12], which correspond to

two types of compressional waves (referred to as P-1 wave and P-2 wave, respectively) and one shear wave (S wave). Among them P-1 wave and S wave are similar to the corresponding ones in elastic theory for ordinary solids, whereas P-2 wave is associated with a diffusion-type process at low frequency due to viscous coupling between solid and fluid phases.

In Fig. 1 the velocities of the three types of waves in a typical sand and soft rock are computed as a function of air proportion ( $1 - S_r$ ) for the frequency of 1 Hz. The properties of the sand and rock used in the calculation are given in Table 1. As can be seen, the velocity of P-1 wave decreases substantially with even a slight decrease below complete saturation. The velocity is maximum for full saturation condition, about 1470 m/s for sand, while it drops to 355 m/s when the degree of saturation is 99.8%. The influence of saturation on S wave is however negligible. The velocity of P-2 wave also reduces with decreasing saturation although the velocity is generally very small at low frequencies due to viscous coupling effect.

2.2. Boundary value problem

The boundary value problem under consideration is illustrated in Fig. 2. An inclined SV wave with angular frequency  $\omega$ , is incident from a half-space of rock media at the base of a multi-layered soil system with a free surface. Both the soil and the rock are modeled here as a partially water-saturated porous medium. As a result, three reflected waves are generated in the half-space; while in any soil layer numbered  $i$ , there are three upward and three downward waves, respectively. For the coordinate system shown, the wave fields in the  $i$ -th layer ( $-\Delta h_i \leq z \leq 0$ ) can be expressed in terms of potentials as follows.

P waves (including downward and upward P-1 wave and P-2 wave):

$$\varphi_s = [A_{11}\exp(-iq_1z) + A_{21}\exp(-iq_2z) + A_{12}\exp(iq_1z) + A_{22}\exp(iq_2z)] \Omega(x, t) \tag{10a}$$

$$\varphi_f = [\delta_1 A_{11}\exp(-iq_1z) + \delta_2 A_{21}\exp(-iq_2z) + \delta_1 A_{12}\exp(iq_1z) + \delta_2 A_{22}\exp(iq_2z)] \Omega(x, t) \tag{10b}$$

SV waves (including downward and upward SV waves):

$$\psi_s = [B_{s1}\exp(-iq_3z) + B_{s2}\exp(iq_3z)] \Omega(x, t) \tag{11a}$$

$$\psi_f = [\delta_3 B_{s1}\exp(-iq_3z) + \delta_3 B_{s2}\exp(iq_3z)] \Omega(x, t) \tag{11b}$$

where  $\Omega(x, t) = \exp[i(\omega t - px)]$  and  $i = \sqrt{-1}$ ;  $\delta_1$ ,  $\delta_2$  and  $\delta_3$  are the amplitude ratios of potentials related to the solid and fluid phases in the porous soil, which can be

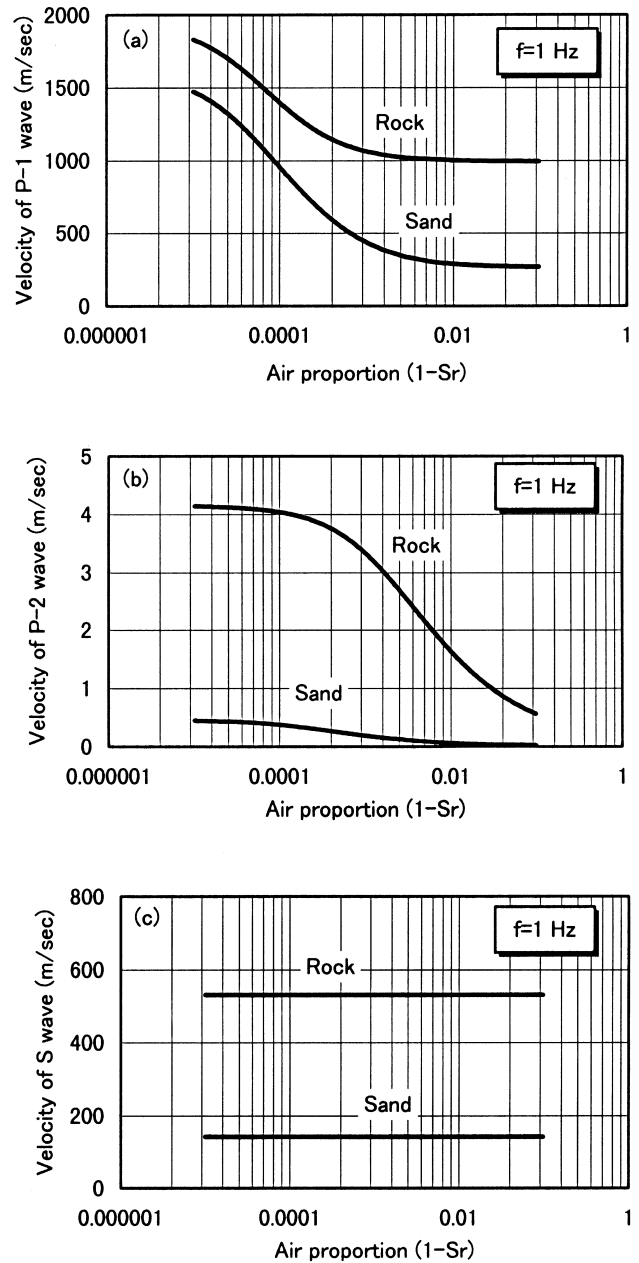


Fig. 1. Effect of saturation on the velocities of three types of waves in a sand and a soft rock.

Table 1  
Properties of the sand and rock used in computation

	Sand	Soft rock
$\rho_s$ Density of solid grains ( $\text{kg/m}^3$ )	2650	2610
$n$ Porosity	0.4	0.3
$\mu$ Shear modulus (MPa)	40	600
$K_b$ Bulk modulus of solid skeleton (MPa)	86.7	1300
$K_s$ Bulk modulus of solid grains (GPa)	36	36
$K_w$ Bulk modulus of pore water (GPa)	2.2	2.2
$\eta$ Viscosity of fluid ( $\text{N s/m}^2$ )	$10^{-3}$	$10^{-3}$
$k$ Permeability ( $\text{m}^2$ )	$10^{-11}$	$10^{-14}$

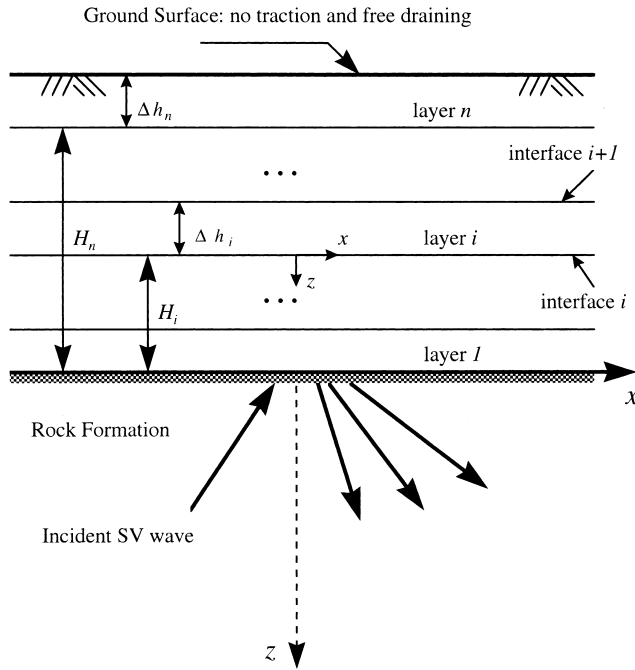


Fig. 2. A multi-layered soil–bedrock system.

determined as [12]

$$\delta_1 = \frac{(\lambda + 2\mu + \alpha^2 M)l_1^2 - \rho\omega^2}{\rho_f\omega^2 - \alpha M l_1^2} \quad (12a)$$

$$\delta_2 = \frac{(\lambda + 2\mu + \alpha^2 M)l_2^2 - \rho\omega^2}{\rho_f\omega^2 - \alpha M l_2^2} \quad (12b)$$

$$\delta_3 = \frac{\mu_s^2 - \rho\omega^2}{\rho_f\omega^2} \quad (12c)$$

in which  $l_1$  and  $l_2$  are the complex wave numbers of P-1 and P-2 waves, respectively, while  $l_s$  is the complex wave number of SV wave.

Based on the relationship between potentials and stresses and displacements, the displacements and stresses in the layer  $i$  can be given in the following convenient form:

$$\begin{Bmatrix} u_x \\ u_z \\ w_z \\ p_f \\ \tau_{xz} \\ \sigma_z \end{Bmatrix}_{-\Delta h_i \leq z \leq 0} = [B]_{6 \times 6} \begin{Bmatrix} A_{11} + A_{12} \\ A_{11} - A_{12} \\ A_{21} + A_{22} \\ A_{21} - A_{22} \\ B_{s1} + B_{s2} \\ B_{s1} - B_{s2} \end{Bmatrix} \quad (13a)$$

or

$$\{Y\}_{-\Delta h_i \leq z \leq 0} = [B]\{A\} \quad (13b)$$

where the elements of the matrix  $[B]$  are given in Appendix. Letting  $z = -\Delta h_i$  and  $z = 0$ , respectively, the relationship

between the quantities at the layer surface and those at the layer base can then be established as

$$\{Y\}_{z=-\Delta h_i} = [B]_{z=-\Delta h_i} [B]_{z=0}^{-1} \{Y\}_{z=0} \quad (14a)$$

or in the following form:

$$\{Y\}_{i+1} = [M]_i \{Y\}_i \quad (14b)$$

in which  $[M]_i = [B]_{z=-\Delta h_i} [B]_{z=0}^{-1}$ .

The relationship derived above may apply to any layers, hence, referring to Fig. 2 and letting  $i = 1, 2, \dots, n$ , one has

$$\begin{aligned} \{Y\}_2 &= [M]_1 \{Y\}_1 \\ \{Y\}_3 &= [M]_2 \{Y\}_2 \\ &\dots \end{aligned} \quad (15)$$

$$\{Y\}_n = [M]_{n-1} \{Y\}_{n-1}$$

$$\{Y\}_{n+1} = [M]_n \{Y\}_n$$

Therefore, the relationship between the quantities at the ground surface and those at the rock base can be given as

$$\{Y\}_{n+1} = [\bar{M}] \{Y\}_1 \quad (16)$$

in which  $[\bar{M}] = [M]_n [M]_{n-1} \dots [M]_2 [M]_1$ .

Similarly, the wave fields in the half-space ( $z \geq 0$ ) can be described as follows.

SV waves (including incident and reflected SV waves):

$$\psi_s = [B_s \exp(-iq'_3 z) + B_i \exp(iq'_3 z)] \Omega(x, t) \quad (17a)$$

$$\psi_f = [\delta'_3 B_s \exp(-iq'_3 z) + \delta'_3 B_i \exp(iq'_3 z)] \Omega(x, t) \quad (17b)$$

P waves (including P-1 wave and P-2 wave):

$$\varphi_s = [A_{s1} \exp(-iq'_1 z) + A_{s2} \exp(-iq'_2 z)] \Omega(x, t) \quad (18a)$$

$$\varphi_f = [\delta'_1 A_{s1} \exp(-iq'_1 z) + \delta'_2 A_{s2} \exp(-iq'_2 z)] \Omega(x, t) \quad (18b)$$

The displacements and stresses in the half-space can then be given in the matrix form as

$$\begin{Bmatrix} u'_x \\ u'_z \\ w'_z \\ p'_f \\ \tau'_{xz} \\ \sigma'_z \end{Bmatrix}_{z \geq 0} = [B']_{6 \times 6} \begin{Bmatrix} A_{s1} \\ A_{s1} \\ A_{s2} \\ A_{s2} \\ B_s + B_i \\ B_s - B_i \end{Bmatrix} \quad (19)$$

in which the matrix  $[B']$  takes the same form with the matrix  $[B]$  in Eqs. (13a) and (13b) except that the corresponding properties related to the rock media are used.

Assuming the stress, displacement, and pore pressure are

continuity at the interface of soil and rock, namely at  $z = 0$ :

$$\begin{Bmatrix} u_x \\ u_z \\ w_z \\ p_f \\ \tau_{xz} \\ \sigma_z \end{Bmatrix}_{z=0^-} = \begin{Bmatrix} u'_x \\ u'_z \\ w'_z \\ p'_f \\ \tau'_{xz} \\ \sigma'_z \end{Bmatrix}_{z=0^+} \quad (20)$$

one may have the following relationship:

$$\{Y\}_1 = [B']_{z=0} \begin{Bmatrix} A_{s1} \\ A_{s1} \\ A_{s2} \\ A_{s2} \\ B_s + B_i \\ B_s - B_i \end{Bmatrix} \quad (21)$$

Note that the ground surface is free of stress and free draining, thus at the surface:

$$\sigma_z = 0, \quad \tau_{xz} = 0, \quad p_f = 0 \quad (22)$$

By enforcing the above conditions and normalizing all coefficients with  $B_i$ , one has

$$[S]_{3 \times 3} \{T\} = \{V\} \quad (23)$$

in which

$$[S]_{3 \times 3} = \begin{bmatrix} D_{41} + D_{42} & D_{43} + D_{44} & D_{45} + D_{46} \\ D_{51} + D_{52} & D_{53} + D_{54} & D_{55} + D_{56} \\ D_{61} + D_{62} & D_{63} + D_{64} & D_{65} + D_{66} \end{bmatrix} \quad (24)$$

$$\{T\} = \begin{Bmatrix} A_{s1} \\ A_{s2} \\ B_s \end{Bmatrix}, \quad \{V\} = \begin{Bmatrix} D_{46} - D_{45} \\ D_{56} - D_{55} \\ D_{66} - D_{65} \end{Bmatrix}$$

with  $D_{ij}$  being the components of the matrix  $[D] = [\bar{M}] [B']_{z=0}$ .

By solving Eq. (23), the stress and displacement in any position in the layer  $i$  can be obtained as

$$\{Y\}_z = [B]_{z=z+H_i} [B]_{z=0}^{-1} [M]_{i-1} [M]_{i-2} \cdots [M]_1 \{Y\}_1 \quad (25)$$

### 3. Numerical examples

The typical sand and soft rock, whose properties have been given in Table 1, are employed in the numerical examples. A special case that represents a half-space made up of the sand will be analyzed firstly to verify the correctness and efficiency of the formulation developed. This is followed by a typical layered model which corresponds a surface sand

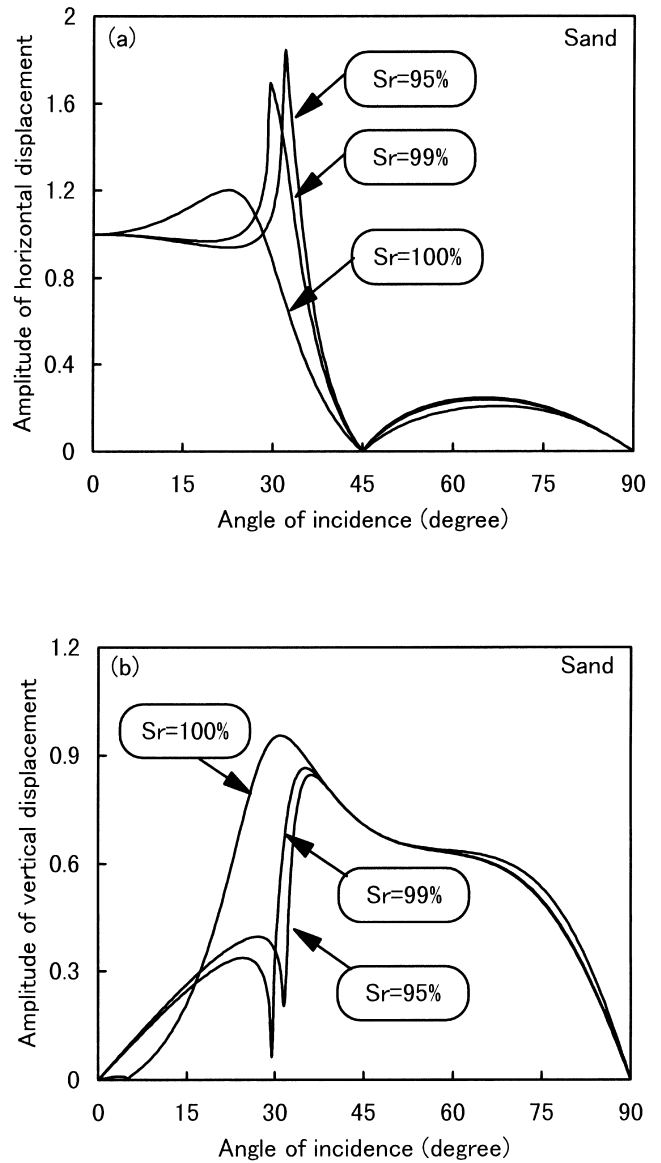


Fig. 3. Effect of saturation on displacement amplitudes in horizontal and vertical components at the surface of a half-space made up of porous sand.

layer overlying a half-space of rock formation. In both examples the effects of saturation will be examined.

#### 3.1. Results for a half-space model

By letting the thickness of the layers vanish, the layered system is reduced to a half-space model. Fig. 3 shows the displacement amplitudes of horizontal and vertical components at the free surface as a function of incident angle for a half-space composed of the porous sand. The frequency is taken as 1 Hz in the computation. Three degrees of saturation are included to show the effect of saturation. In order to simultaneously illustrate the influence of incident angle, both the horizontal and vertical amplitudes are normalized by the amplitude due to vertically incident SV wave. Fig. 4 shows the results for the displacement ratios

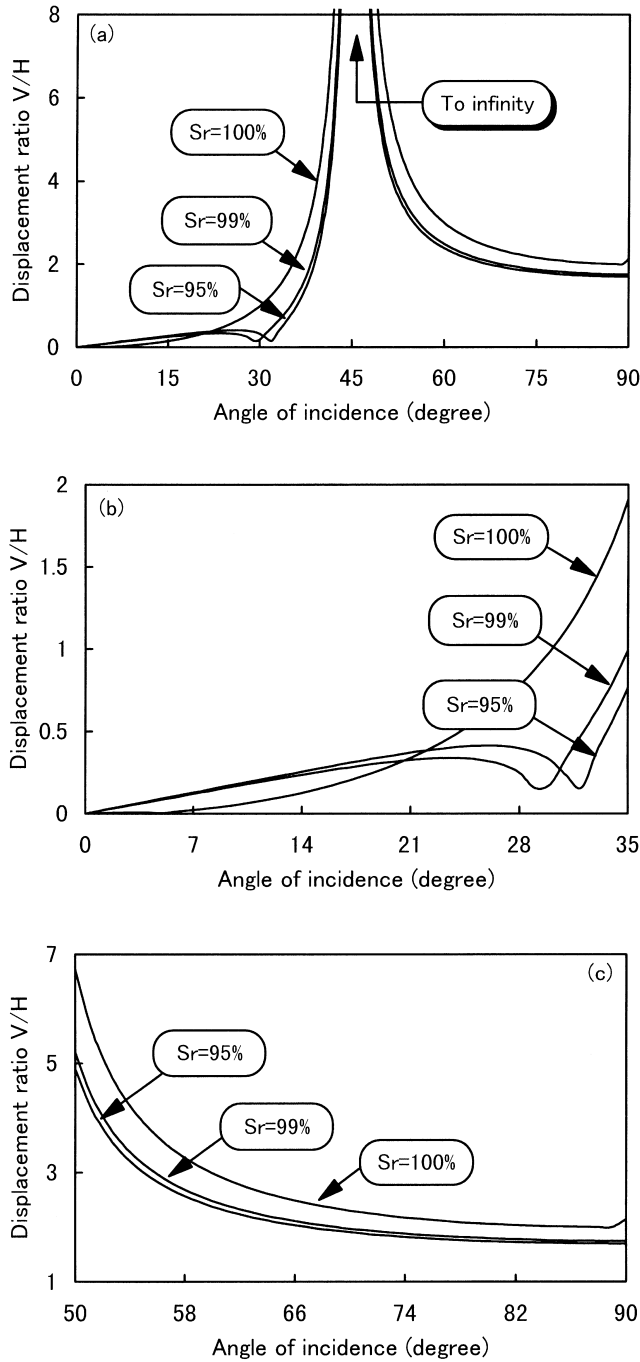


Fig. 4. Effect of saturation on vertical-to-horizontal displacement ratios at the surface of a half-space.

of vertical to horizontal component at the free surface as a function of angle of incidence for the three cases of saturation.

It is found that these results are in general similar to the classic results for a viscoelastic solid half-space [8,13]. For all the cases of saturation considered, the displacement in the horizontal component is zero at the angle of 45°, which leads to a singularity at this point in the vertical-to-horizontal ratios. The amplitudes of hori-

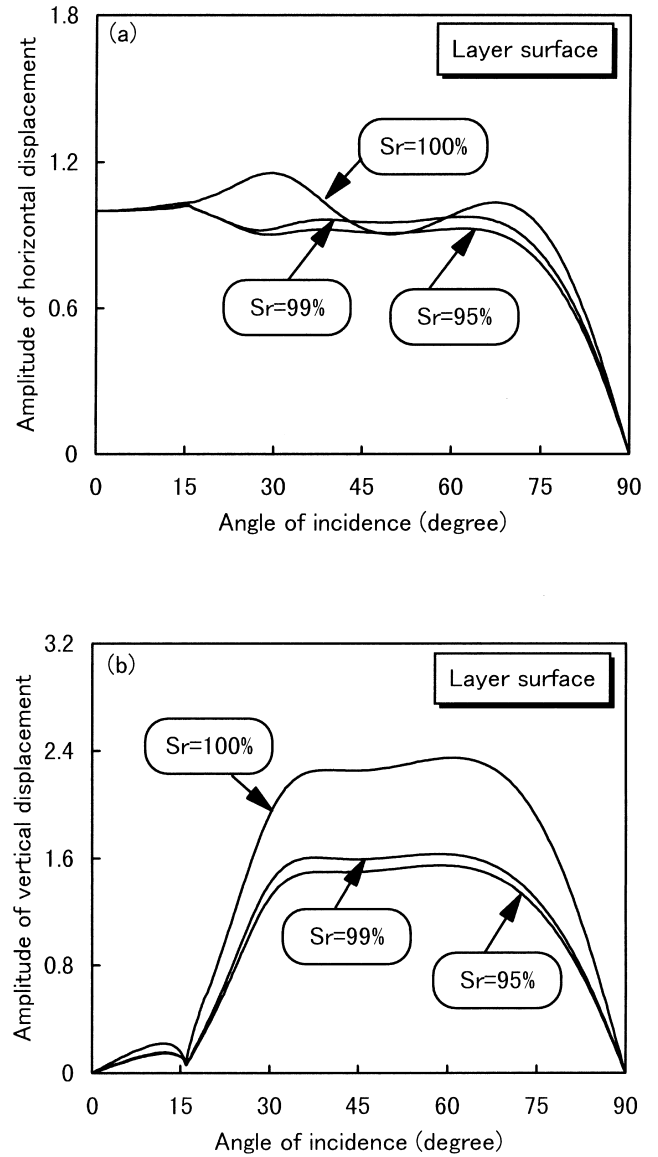


Fig. 5. Effect of saturation on displacement amplitudes in horizontal and vertical components at layer surface.

zontal displacements due to inclined SV wave are smaller than those due to vertical SV wave when the angles of incidence are greater than about 40°. However, in a narrow range of angles (i.e. between 20 and 35°), a peak forms with the peak value significantly exceeding 1 for any case of saturation considered. It is of interest to note that substantial difference exists in this range between the full saturation case and the partial saturation case. Even a slight decrease of full saturation can cause a much sharper peak with a higher peak value, whereas the vertical component at this point forms a downward cusp.

The significant influence of saturation also exist in the displacement ratios between the vertical and horizontal components, as demonstrated by Fig. 4. The degree of the influence is also dependent on the angle of incidence.

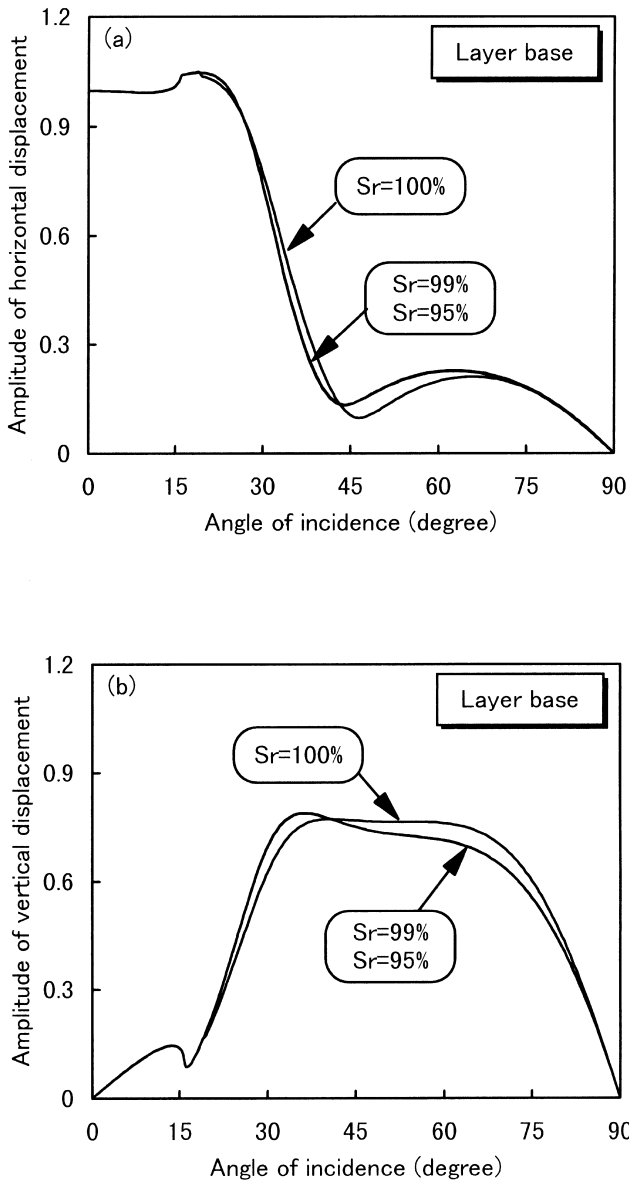


Fig. 6. Effect of saturation on displacement amplitudes in horizontal and vertical components at layer base.

### 3.2. Results for a surface layer model

Next, a surface sand layer of 20 m overlying a half-space of rock media is considered. The fundamental fixed-base frequency of the layer is about 1.78 Hz. The frequency of the incident SV wave is taken as 1 Hz, which corresponds to a shear wave length of 142 m in the layer. In the computation the rock media is assumed to be always fully water-saturated while the degree of saturation of the surface sand varies from complete saturation to partial saturation. Fig. 5(a) and (b) shows, respectively, the amplitudes of horizontal and vertical displacements at the layer surface as a function of incident angle for three cases of saturation. It is seen again that the effect of saturation is profound on the amplitudes in both components, especially on the vertical components; and

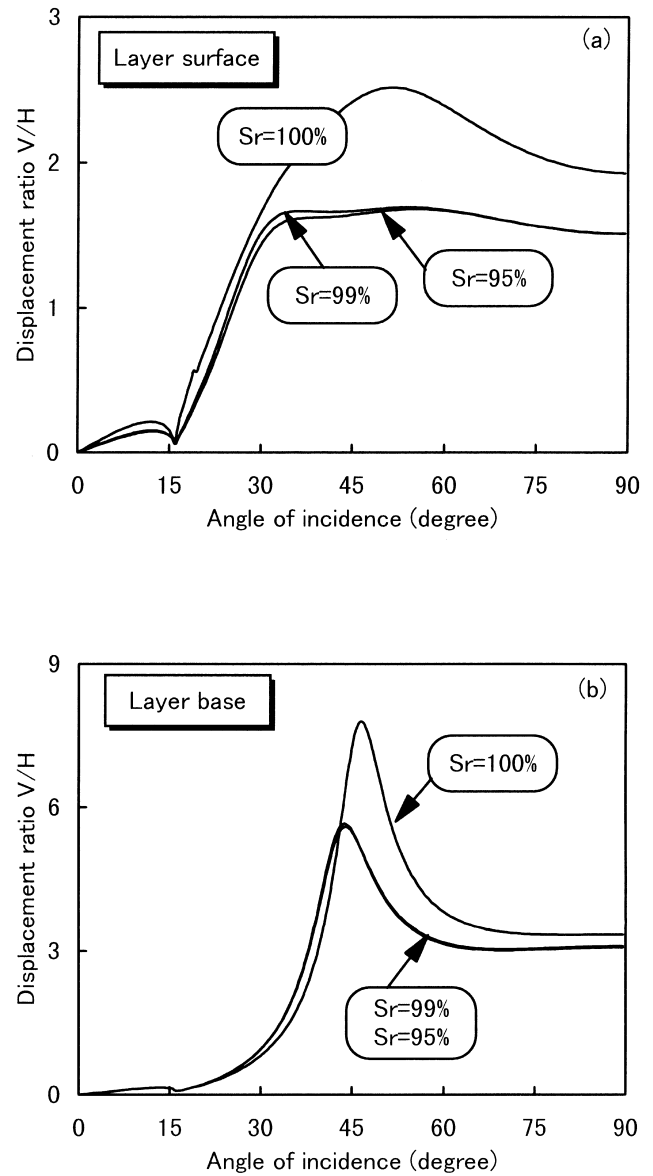


Fig. 7. Effect of saturation on vertical-to-horizontal displacement ratios at layer surface and layer base.

the effect strongly depends on the angle of incidence. Even a slight decrease of full saturation may cause a substantial difference in the amplitudes at some angles. Whereas for the ground motion at the layer base, the influence of saturation becomes weak in both components when compared to that at the layer surface, as shown in Fig. 6.

The results for the ratios of vertical to horizontal component (V/H) at the layer surface and layer base are shown in Fig. 7(a) and (b), respectively, as a function of angle of incidence and for the three cases of saturation considered. Generally, the ratios at the surface exhibit a different behavior from that at the layer base. For the ratios either at the surface or at the base, it is found that, a decrease of full saturation may lead to a great change in the ratios when the SV wave is incident with large angles, whereas for

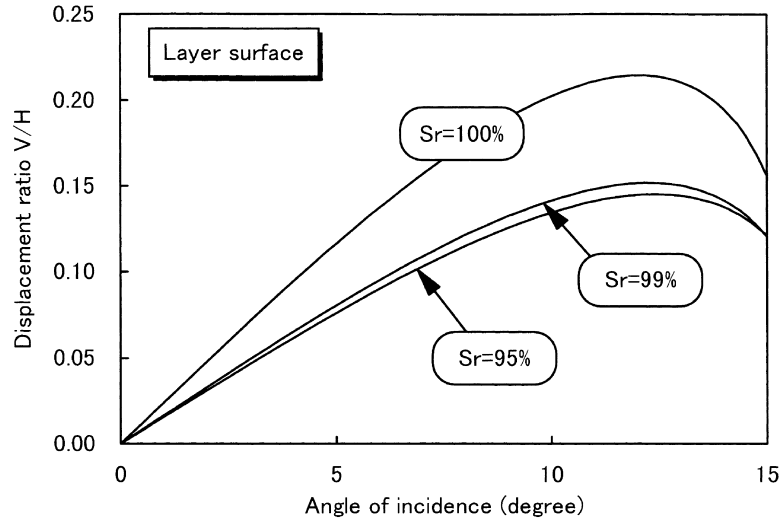


Fig. 8. Effect of saturation on displacement ratios at layer surface at small angles of incidence.

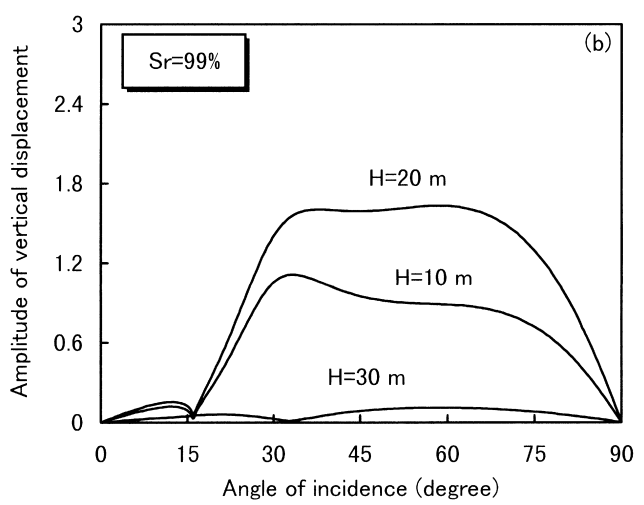
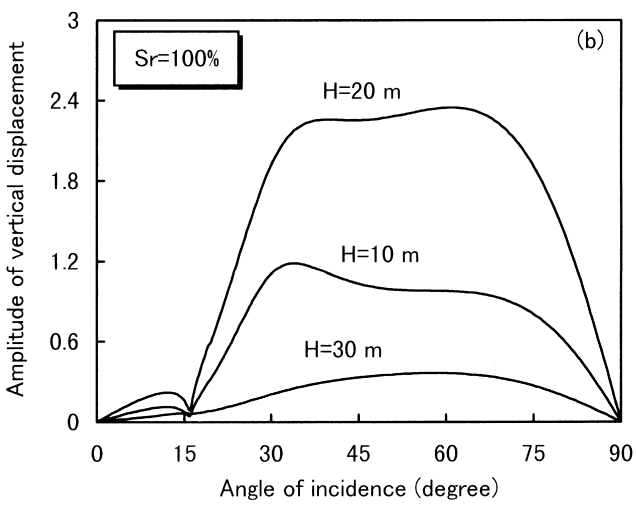
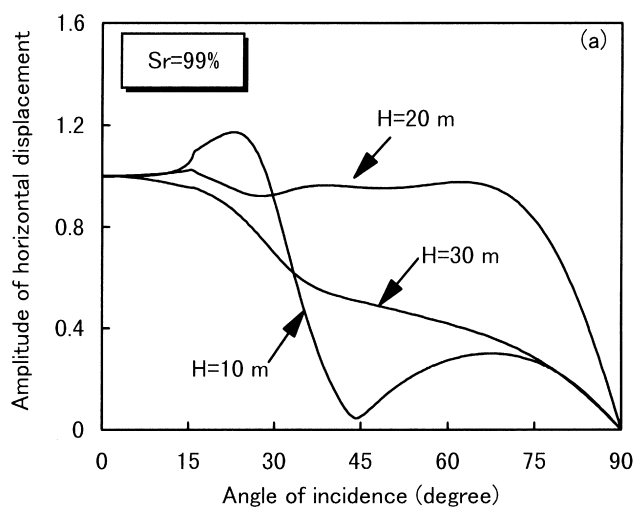
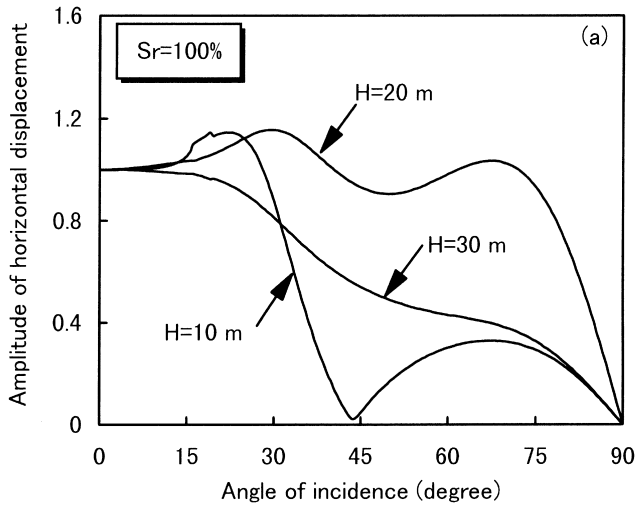


Fig. 9. Effect of layer thickness on angle-dependent displacement amplitudes ( $S_r = 100\%$ ): (a) horizontal component; and (b) vertical component.

Fig. 10. Effect of layer thickness on angle-dependent displacement amplitudes ( $S_r = 99\%$ ): (a) horizontal component; and (b) vertical component.



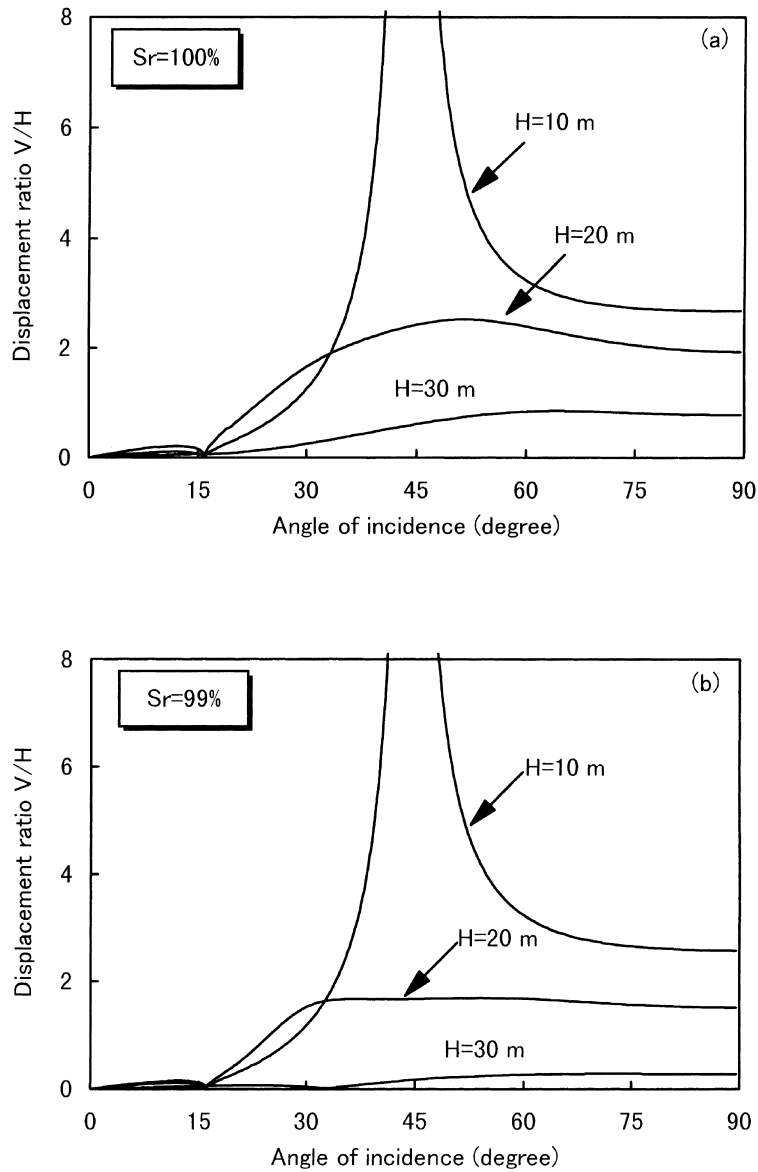


Fig. 11. Effect of layer thickness on angle-dependent displacement ratios: (a)  $S_r = 100\%$ ; and (b)  $S_r = 99\%$ .

small angles of incidence the influence of saturation is relatively slight. However, it is to be noted that, even if the SV wave is incident with a small angle (e.g.  $3^\circ$ ), the discrepancy in the amplitude ratios  $V/H$  between the full and partial saturation conditions at the ground surface may still be great in the sense of relativity, as demonstrated by Fig. 8.

In order to investigate the effect of thickness (or implicitly the effect of frequency), two additional cases of geometry, a surface layer of 10 and 30 m, respectively, are analyzed. Figs. 9 and 10 show the angle-dependent amplitudes in both horizontal and vertical components at the ground surface for the three cases of thickness in full and partial saturation conditions, respectively. Fig. 11 demonstrates the corresponding angle-dependent displacement ratios as affected by the degree of saturation. It is noted that the influence of layer thickness is great on the results

in both full and partial saturation conditions. This is mainly because that the fundamental frequencies for the layered systems corresponding to the three cases of thickness are 3.55, 1.78 and 1.18 Hz, respectively.

#### 4. Conclusions

A study of the saturation effects on the horizontal and vertical components of ground motions in a multi-layered soil–bedrock system has been described. By treating the soil and the underlying rock as a partially water-saturated porous medium, an efficient formulation was developed for the computation of the two-dimensional ground motions due to an obliquely incident SV wave.

Numerical results for both the half-space model and the

single-layered model indicate that the influence of saturation may be significant and this influence is dependent on the angle of incidence. Even a slight decrease of full saturation of the overlying soil may cause appreciable difference in the amplitudes of ground motions in both the horizontal and vertical components and the amplitude ratios between the two components at the ground surface, suggesting that one may need to carefully take into account the saturation conditions in the interpretation of field observations, especially in such situations that partial saturation may very probably take place (e.g. for offshore soils and marine sediments).

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### Appendix A

The components of the matrix  $[B]$  in Eqs. (13a) and (13b) are as follows:

$$B_{11} = -ip\cos(q_1z) \quad B_{12} = -p\sin(q_1z) \quad B_{13} = -ip\cos(q_2z)$$

$$B_{14} = -p\sin(q_2z) \quad B_{15} = q_3\sin(q_3z) \quad B_{16} = iq_3\cos(q_3z)$$

$$B_{21} = -q_1\sin(q_1z) \quad B_{22} = -iq_1\cos(q_1z) \quad B_{23} = -q_2\sin(q_2z)$$

$$B_{24} = -iq_2\cos(q_2z) \quad B_{25} = -ip\cos(q_3z) \quad B_{26} = -p\sin(q_3z)$$

$$B_{31} = -\delta_1q_1\sin(q_1z) \quad B_{32} = -i\delta_1q_1\cos(q_1z) \quad B_{33} = -\delta_2q_2\sin(q_2z)$$

$$B_{34} = -i\delta_2q_2\cos(q_2z) \quad B_{35} = -i\delta_3p\cos(q_3z) \quad B_{36} = -\delta_3p\sin(q_3z)$$

$$B_{41} = l_1^2(\delta_1M + \alpha M)\cos(q_1z) \quad B_{42} = -il_1^2(\delta_1M + \alpha M)\sin(q_1z)$$

$$B_{43} = l_2^2(\delta_2M + \alpha M)\cos(q_2z) \quad B_{44} = -il_2^2(\delta_2M + \alpha M)\sin(q_2z)$$

$$B_{45} = 0 \quad B_{46} = 0$$

$$B_{51} = 2\mu pq_1 i\sin(q_1z) \quad B_{52} = -2\mu pq_1 \cos(q_1z)$$

$$B_{53} = 2\mu pq_2 i\sin(q_2z) \quad B_{54} = -2\mu pq_2 \cos(q_2z)$$

$$B_{55} = \mu(q_3^2 - p^2)\cos(q_3z) \quad B_{56} = -\mu(q_3^2 - p^2)i\sin(q_3z)$$

$$B_{61} = -[(\lambda + \alpha^2M + 2\mu + \delta_1\alpha M)l_1^2 - 2\mu p^2]\cos(q_1z)$$

$$B_{62} = i[(\lambda + \alpha^2M + 2\mu + \delta_1\alpha M)l_1^2 - 2\mu p^2]\sin(q_1z)$$

$$B_{63} = -[(\lambda + \alpha^2M + 2\mu + \delta_2\alpha M)l_2^2 - 2\mu p^2]\cos(q_2z)$$

$$B_{64} = i[(\lambda + \alpha^2M + 2\mu + \delta_2\alpha M)l_2^2 - 2\mu p^2]\sin(q_2z)$$

$$B_{65} = 2\mu pq_3 i\sin(q_3z)$$

$$B_{66} = -2\mu pq_3 \cos(q_3z)$$

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