Evaluating Liquefaction Strength of Partially Saturated Sand

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Abstract: A method is presented for evaluating the liquefaction strength of partially saturated sand using the compression wave velocity (*P*-wave velocity), a new indicator of saturation. Based on laboratory test results, an empirical correlation that relates the liquefaction strength with the pore pressure coefficient *B* is firstly proposed. The strength is defined as the cyclic stress ratio required to cause liquefaction at a specified number of cycles. With the aid of a theoretical relation between *B* and the *P*-wave velocity, an explicit correlation of more interest is then established between the liquefaction strength of sand and its *P*-wave velocity. A comparison of the predictions using this explicit correlation with laboratory measurements shows a satisfactory agreement. The significance of this method lies in that it makes it possible to evaluate the liquefaction strength of sand as affected by saturation through the measurement of *P*-wave velocity, which can be made not only in the laboratory but particularly in the field.

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Introduction

In certain situations soils below the groundwater table are not, as usually assumed, fully saturated. The condition of partial saturation may be caused by, for example, fluctuating water tables that are associated with natural or manmade processes. In earthquake geotechnical engineering, the partial saturation condition may give rise to two major impacts. One is that partial saturation of soil can cause much greater amplification in vertical ground motion compared to a fully saturated model, as demonstrated by a detailed study on a well-documented case from the 1995 Kobe earthquake (Yang and Sato 2000, 2001). The other impact is related to the liquefaction resistance of sandy soils. Laboratory tests (Sherif et al. 1977; Chaney 1978; Yoshimi et al. 1989) have shown that the liquefaction resistance of sands depends strongly on the degree of saturation, which was expressed in terms of the pore pressure coefficient *B* (Skempton 1954). At a specified cyclic stress ratio, the number of cycles causing liquefaction was found to increase substantially with decreasing values of *B*.

The *B*-value method has been widely employed to evaluate the state of saturation of laboratory samples. However, utilizing this method in the field to determine in situ states of saturation is apparently difficult. As a result, a further development of approaches for evaluating in situ liquefaction strength that take into account the saturation effect is hampered, although this effect has long been recognized. An alternative method that can be employed conveniently in both laboratory and field to determine the saturation state of soil is therefore desirable.

The seismic method through measuring the velocity of compression waves (i.e., *P*-waves) appears to meet the desire. It has been known that the *P*-wave velocity is sensitive to even a slight decrease of full saturation (Richart et al. 1970) and hence holds a potential as an indicator of saturation. The velocity of *P*-waves can be measured in the laboratory using bender elements attached to the samples (e.g., Nakagawa et al. 1996; Lings and Greening 2001) and in the field using the crosshole or downhole technique. Effectiveness of the use of *P*-wave velocity and Poisson's ratio in identifying in situ partial saturated zones has been demonstrated by a borehole array site (Yang and Sato 2000).

Aimed at characterizing the saturation effects on the liquefaction potential of sand in both the field and laboratory, Yang (2002) recently proposed an approach in which an explicit relationship was developed between the liquefaction resistance of sand and its *P*-wave velocity. This relationship was obtained by introducing a theoretical relationship between the value of *B* and the *P*-wave velocity into an empirical correlation relating the liquefaction resistance to *B*, established based on some laboratory test data. The liquefaction resistance in question was expressed in terms of the number of cycles at a specified cyclic stress ratio to cause liquefaction while the analyzed data was from two series of torsional shear tests on standard sands (Sherif et al. 1977; Yoshimi et al. 1989).

Since the liquefaction resistance of sand is more preferably defined in engineering practice as the cyclic stress ratio required to cause liquefaction at a specified number of cycles, an extension of the original work is made herein by adopting this definition. A new explicit relationship is established between the liquefaction strength of sand in terms of the normalized cyclic stress ratio at 20 cycles and its *P*-wave velocity. The relationship is developed following the concept given by Yang (2002) yet based on a wider database which includes two additional series of cyclic triaxial

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Table 1. Sand Properties and Test Description

Material	D_{50} (mm)	C_u	$e_{\max}$	$e_{\min}$	D_r	Liquefaction criteria	Test type	Reference
Ottawa sand	0.40	2.1	0.76	0.50	—	EPP$=\sigma_c'$	Torsional shear	Sherif et al. (1977)
Toyoura sand	0.175	1.52	0.976	0.605	60%	DA$=$5%	Torsional shear	Yoshimi et al. (1989)
Tongjiazhi sand	0.1	3.7	—	—	60%	EPP$=\sigma_c'$	Triaxial	Xia and Hu (1991)
Niigata sand	0.325	1.47	—	—	62%	DA$=$5%	Triaxial	Ishihara et al. (2001)

Note: DA=double amplitude of strain; EPP=excess pore pressure; and σ_c'=initial confining pressure.

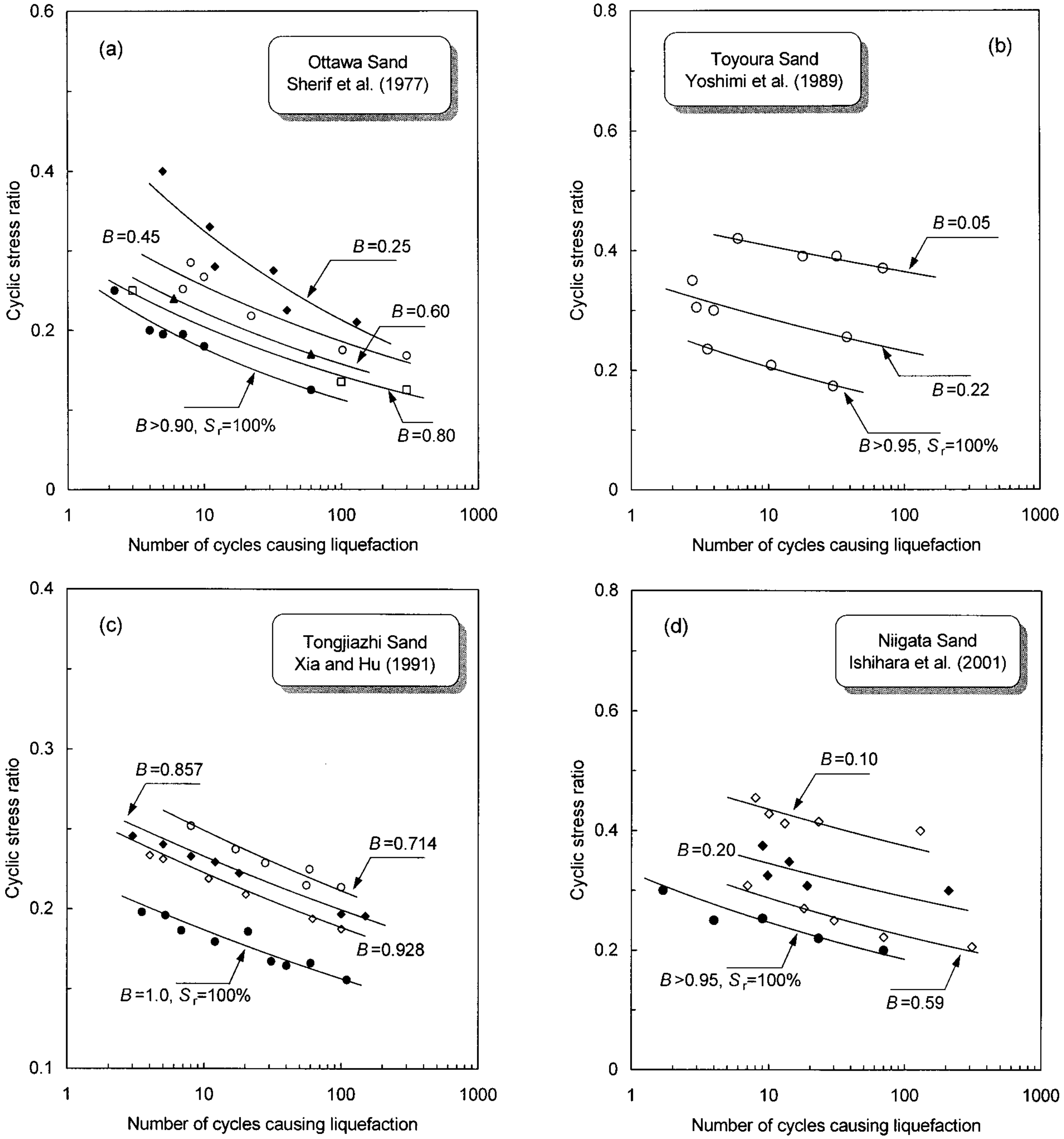

Fig. 1. Laboratory test data for liquefaction strength of sands affected by saturation: (a) Ottawa sand; (b) Toyoura sand; (c) Tongjiazhi sand; and (d) Niigata sand

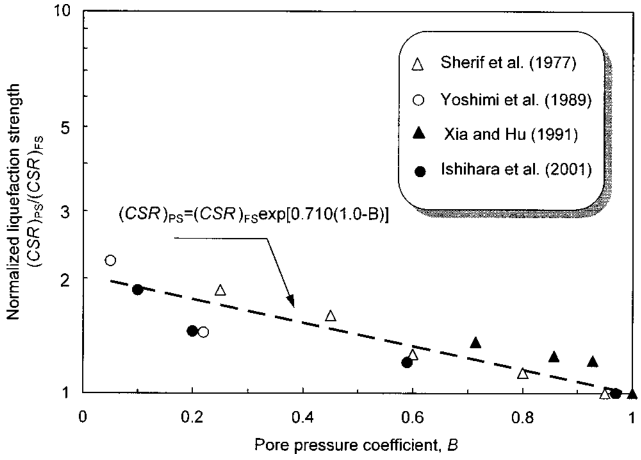

Fig. 2. Normalized liquefaction strength as a function of pore pressure coefficient B

test results (Xia and Hu 1991; Ishihara et al. 2001). Since some of the newly included tests (Ishihara et al. 2001) involved with the measurement of P-wave velocity, validation of the theoretical developments is further made through a comparison of the predictions with the measurements.

Liquefaction Strength in Relation to Pore Pressure Coefficient

Four series of cyclic tests as summarized in Table 1 are investigated. Test results under different values of pore pressure coefficient, B, are shown in Fig. 1, in which the cyclic stress ratio is plotted against the number of cycles causing liquefaction. It is clear that the liquefaction resistance of sand increases considerably as the value of B decreases or correspondingly the degree of saturation decreases. In order to characterize the saturation effects observed liquefaction strength is defined herein as the cyclic stress ratio required to cause liquefaction at 20 cycles. When the liquefaction strength at lower values of B is normalized by that at the largest value of B (i.e., full saturation condition), the normalized strength plotted on a log scale shows a good linear correlation with B, as shown in Fig. 2. The test results can be fitted reasonably well by a dashed line that is generated using the following empirical function:

$$(\mathrm{CSR})_{\mathrm{PS}}=(\mathrm{CSR})_{\mathrm{FS}}e^{[0.710(1.0-B)]} \qquad (1)$$

in which $(\mathrm{CSR})_{\mathrm{PS}}$ denotes the cyclic stress ratio required to cause liquefaction at 20 cycles under a partially saturated condition and $(\mathrm{CSR})_{\mathrm{FS}}$ is the required cyclic stress ratio at full saturation.

The empirical correlation may be given in a more general form as

$$(\mathrm{CSR})_{\mathrm{PS}}=(\mathrm{CSR})_{\mathrm{FS}}e^{[\alpha(1.0-B)]} \qquad (2)$$

in which α is a parameter to be calibrated from test results and may further be considered as a function of the potential factors that might affect the correlation, such as the number of cycles used for defining the liquefaction strength.

Relation between Pore Pressure Coefficient and *P*-Wave Velocity

The details for the development of the relation between the P-wave velocity and the pore pressure coefficient, B, are referred to in Yang (2002) and the references therein. In what follows only some highlights are given.

The pore pressure coefficient B can be related to the degree of saturation S_r in the following form:

$$B=\frac{1}{1+n\dfrac{K_b}{K_w}+n\dfrac{K_b}{P_a}(1-S_r)} \qquad (3)$$

in which S_r = degree of saturation; n = porosity; p_a = absolute fluid pressure; K_b = bulk modulus of soil skeleton; and K_w = bulk modulus of pore water.

The velocity of P-waves in a partially saturated soil with incompressible solid grains can be determined by

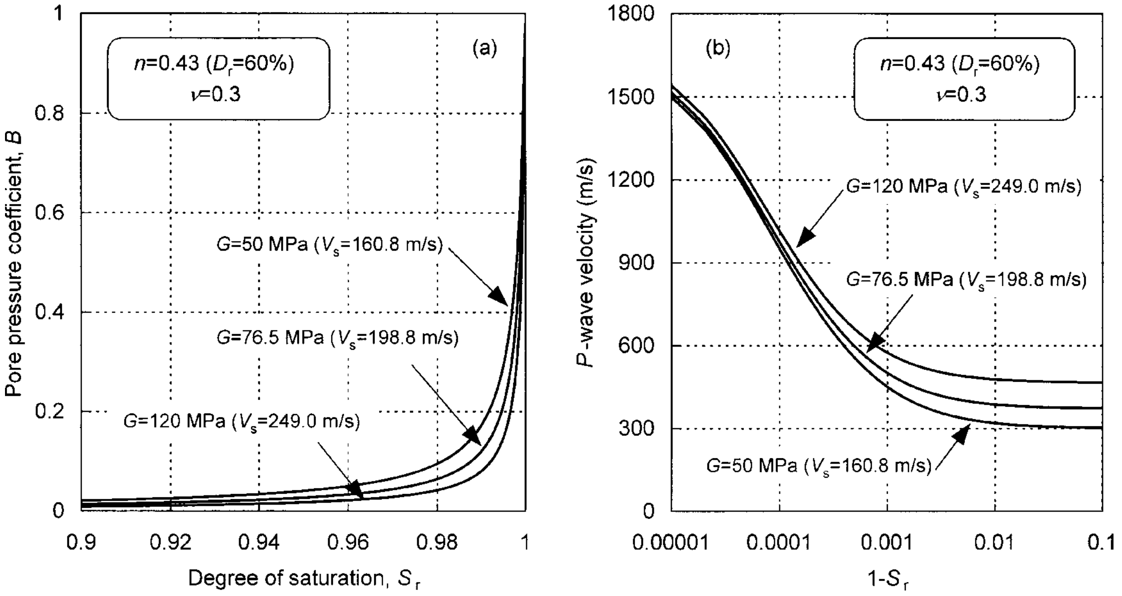

Fig. 3. Variations of (a) pore pressure coefficient and (b) P-wave velocity with degree of saturation

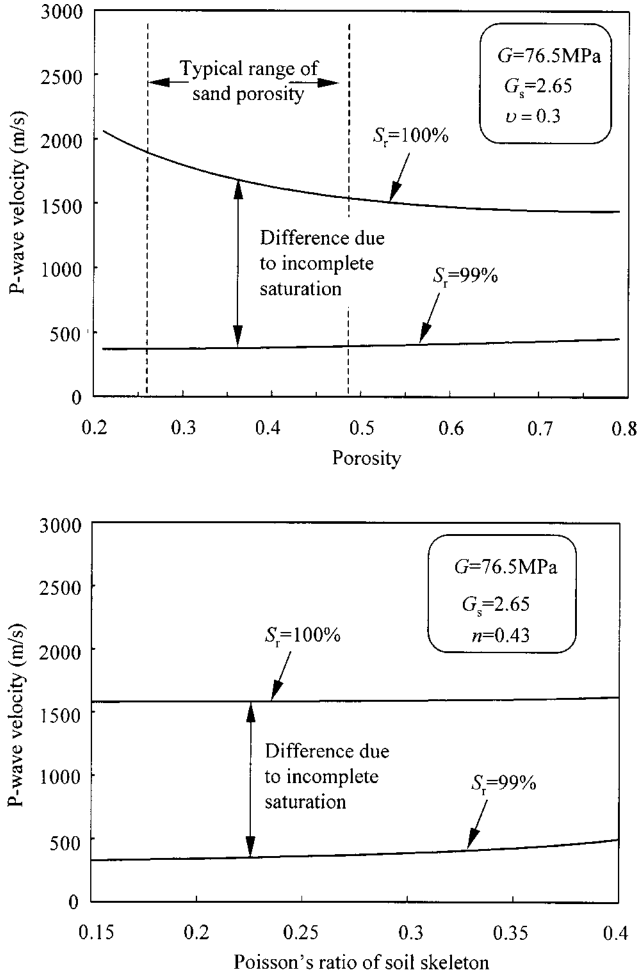

Fig. 4. Effects of porosity and Poisson's ratio on *P*-wave velocity

$$V_P=\left(\frac{K_b+\dfrac{4G}{3}+\dfrac{K_w}{\vartheta n}}{(1-n)\rho_s+n\rho_f}\right)^{1/2} \tag{4}$$

in which G = shear modulus; ρ_s = mass density of grains; ρ_f = density of pore fluid; and $\vartheta = 1+K_w(1-S_r)/p_a$.

From Eqs. (3) and (4) an expression relating the *P*-wave velocity with B can readily be achieved as follows:

$$V_P=\left(\frac{\dfrac{4G}{3}+\dfrac{K_b}{1-B}}{(1-n)\rho_s+n\rho_f}\right)^{1/2} \tag{5}$$

Based on Eqs. (3) and (4), the values of B and the *P*-wave velocity for Toyoura sand are computed as a function of degree of saturation and shown in Fig. 3. As indicated in Yoshimi et al. (1989), the shear modulus G is taken as 76.5 MPa and the porosity of the sand is taken as 0.43 (corresponding to the relative density of 60%). The bulk modulus of the solid skeleton is determined by $K_b = 2G(1+\nu)/3(1-2\nu)$ where the Poisson's ratio ν is assumed as 0.3. The absolute pressure is taken as being the atmospheric pressure. In order to show the influence of soil stiffness, two additional cases of shear modulus, G = 50 and 120 MPa, are included in the same graphs.

It is clear from Fig. 3 that both B and the *P*-wave velocity are sensitive to a change of saturation. At full saturation B approaches 1, while the *P*-wave velocity wave is over 1,500 m/s; but a decrease of complete saturation to 90% may lead to a value of B close to 0.01 with a *P*-wave velocity as low as 370 m/s. Fig. 3 also indicates that at a specified value of B, a stiff soil may achieve a higher degree of saturation than a soft soil.

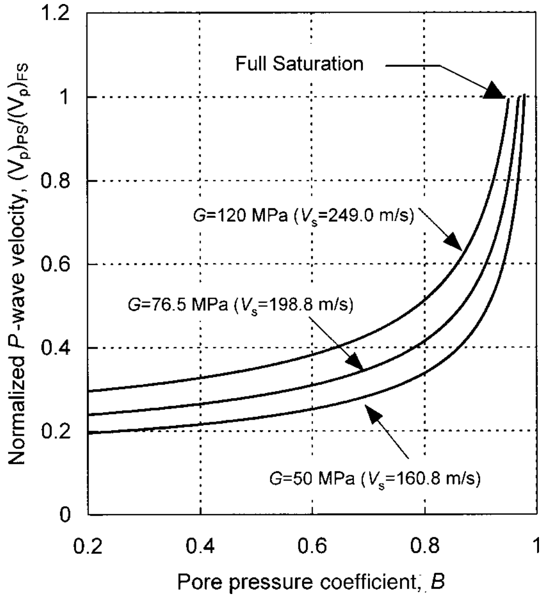

Fig. 5. Relationship between *P*-wave velocity and *B*

To identify the potential effect of sand density and Poisson's ratio ν on the *P*-wave velocity, a wide range of values of porosity and Poisson's ratio are employed in the computation under otherwise identical conditions for two cases of saturation, 100 and 99%, respectively. The range of porosity corresponds to a wide range of density, from about 1,300 to 2,300 kg/m^3. The results shown in Fig. 4 indicate that, compared to the effect of incomplete saturation, the influence of varying density/porosity and Poisson's ratio is very small.

Using Eq. (5), the relationship between the *P*-wave velocity and B for the three cases of shear modulus is illustrated in Fig. 5, where the *P*-wave velocity under partial saturation conditions is normalized by that under full saturation conditions. It is evident from Fig. 5 that the *P*-wave velocity increases as the value of B increases while for a specific value of B the normalized velocity increases with soil stiffness.

Liquefaction Strength Affected by *P*-Wave Velocity

Based on the empirical function established between the liquefaction strength and B, Eq. (1) or (2), and the theoretical relation between B and *P*-wave velocity, an explicit correlation can be developed relating the liquefaction strength of sand with its *P*-wave velocity as follows, by a straightforward algebraic manipulation:

$$(\mathrm{CSR})_{\mathrm{PS}}=(\mathrm{CSR})_{\mathrm{FS}}e^{\{0.710R/[(V_p/V_s)^2-4/3]\}} \tag{6}$$

or

$$(\mathrm{CSR})_{\mathrm{PS}}=(\mathrm{CSR})_{\mathrm{FS}}e^{\{\alpha R/[(V_p/V_s)^2-4/3]\}} \tag{7}$$

in which $R=[2(1+\nu)]/[3(1-2\nu)]$ and V_s = velocity of shear waves.

The explicit relationship expressed in Eq. (6) or (7) is of considerable interest because it makes it possible to evaluate the liq-

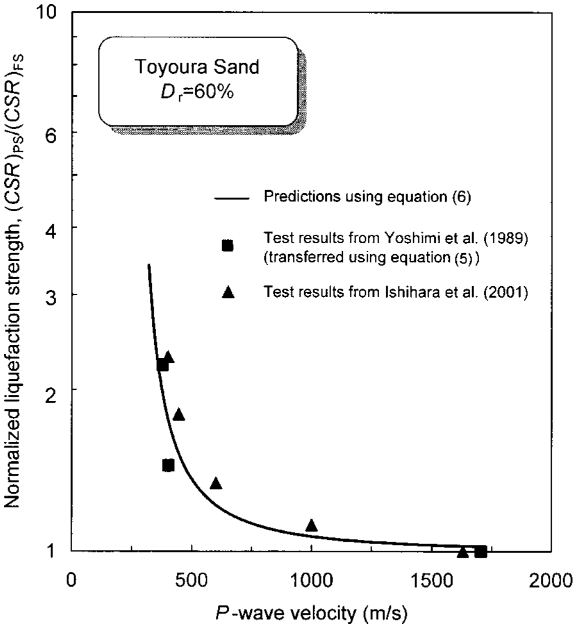

Fig. 6. Normalized liquefaction strength as a function of *P*-wave velocity for Toyoura sand

uefaction strength of sands as affected by saturation through the measurements of *P*-wave velocity, which can be made not only in the laboratory but particularly in the field by means of the cross-hole or downhole technique. Using this relationship Fig. 6 shows the normalized liquefaction strength against the *P*-wave velocity for Toyoura sand. Obviously, the liquefaction strength correlates with the velocity of *P*-waves. The strength will be increased by a factor of 2 when the *P*-wave velocity decreases from about 1,600 to 400 m/s. For the sake of comparison, two sets of test results for Toyoura sand at the same relative density are included in the figure. One set is that due to Yoshimi et al. (1989), which is obtained by transferring the results in terms of B with Eq. (5). The other set is from Ishihara et al. (2001), who performed the measurements of *P*-wave velocity in the samples before cyclic testing. It is evident that the predictions using the relationship derived agree reasonably well with the test results.

Concluding Remarks

An approach has been suggested for evaluating the saturation effects on the liquefaction strength of sand. Based on laboratory test data, an empirical correlation between the liquefaction strength and the pore pressure coefficient B was proposed. The strength in question was defined as the cyclic stress ratio required causing liquefaction at 20 cycles. With the aid of a theoretical relation between B and the *P*-wave velocity, an explicit correlation of more interest was then established between the liquefaction strength of sand and its *P*-wave velocity. The comparison of the predictions with laboratory measurements showed a satisfactory agreement. Since the *P*-wave velocity can be measured in both the field and laboratory, the proposed method may allow for an efficient characterization of saturation effects on the liquefaction strength of sand.

Last but not least, it should be mentioned that various potential factors remain to be investigated within the proposed framework and the established correlation can be improved with more quality test data becoming available.

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Notation

The following symbols are used in this technical note:

B = pore pressure coefficient;
$(\mathrm{CSR})_{\mathrm{FS}}$ = cyclic stress ratio required causing liquefaction at 20 cycles in full saturation;
$(\mathrm{CSR})_{\mathrm{PS}}$ = cyclic stress ratio required causing liquefaction at 20 cycles in partial saturation;
D_r = relative density;
G = shear modulus;
G_s = specific gravity of grains;
K_b = bulk modulus of soil skeleton;
K_f = bulk modulus of pore fluid;
K_w = bulk modulus of pore water;
n = porosity;
p_a = absolute pore pressure;
S_r = degree of saturation;
V_p = velocity of compression waves (*P*-waves);
V_s = velocity of shear waves (*S*-waves);
ν = Poisson's ratio;
ρ = total density;
ρ_f = density of pore fluid; and
ρ_s = density of solid grains.

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